Population Fluctuations in Ecosystems

New Mexico

Supercomputing Challenge

Final Report

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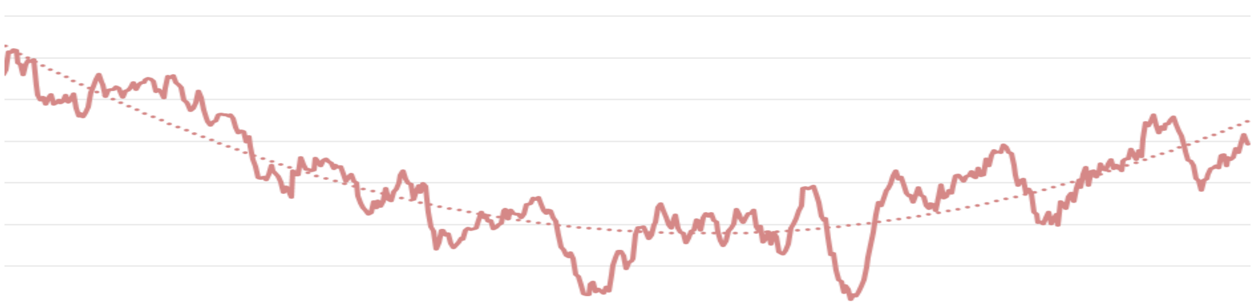
Team 4

Albuquerque Academy



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1.0 Executive Summary

The interactions between organisms are the driving force for life on Earth. Some of these interactions include the consumption of one organism by another, reproduction between two members of the same species, and behaviors in movement. In the broadest terms, we can classify organisms by their means of attaining nutrients and energy. Producers are organisms that get energy from a direct source, be it through photosynthesis or chemosynthesis. Consumers are organisms with a method of locomotion that gain energy by eating other organisms. Primary consumers are organisms which consume producers, whereas secondary consumers, or predators, consume primary consumers, producers, or other secondary consumers.

Ecosystems are groups of organisms; analyzing an ecosystem means trying to understand the different interactions that occur between the organisms it encompasses. In this project, a set of behaviors for the member organisms will allow us to see the overall behavior of the ecosystem. Our algorithm models three different species of organisms: predators, primary consumers, and plants (producers). In the report, we will refer to the predators, primary consumers, and plants as dinosaurs, bigfeet, and plants respectively. These species all behave in unique ways, representing animals and plants that we see on Earth. The algorithm we created uses an agent-based model to help us understand how wild animal populations fluctuate; the algorithm also delves into the effects of cataclysmic events. Understanding the effects of human interaction upon a group of organisms will grow increasingly important in this millennia; learning the differences between harmless, detrimental, and catastrophic interactions may help protect the biodiversity we see on this planet.

2.0 Problem Statement

What intrinsic traits of an animal have the largest effect on the entire ecosystem? Our model proposes an agent based model rather than existing equations to solve this. At first, we were interested in looking at making each animal with different traits, but we observed the data in our current model would be more meaningful than adding traits. We wanted to know what happened to populations of each species over time. Would every run result in the extinction of the predator species given enough time? Our goal was to provide an accurate prediction of how species would change over time using an agent based model.

3.0 Background Information

During World War I, fishing in the Mediterranean was sharply reduced to help support the total war effort. When fishing resumed after the war ended, the fish population had been decimated. A scientist named Vito Volterra attempted to understand the cause of this enigma. He deduced that after the fish population was left unchecked, the shark population began to thrive. Once the shark population increased, it ravaged the local fish population. Volterra learned that populations follow cyclical patterns of peaks and troughs; the fisherman resumed while the fish population was experiencing a trough. Volterra also learned that the removal of a member of an ecosystem greatly impacts the surrounding populations, destabilizing an ecosystem.

Simultaneous but exclusively, Volterra, and an American scientist, Alfred Lotka, developed an equation which could help explain this change in populations. This equation would later be known as the Lotka-Volterra equation. The first equation shows that a true equilibrium is never established in an animal populations; the populations will rise and fall in correlation to the abundance of their food. This equation works well for the ecosystem that we are modeling, where there is a plethora of food and the predator only relies on one (or very similar) species of prey. The basic equation changes when food is not so plentiful so the prey are spread out. A new term is added that involves the capacity and density of the prey, so there is no result in a situation in which the predators can survive sustainably. This explains why places like Australia have an ecosystem in which the greatest amount of predators are large reptiles, like snakes, because they have greater efficiency in extracting energy from their food. See figure 5.00 for a graphical demonstration of this concept.

The Lotka-Volterra equations can be easily modified to model much more than just predators and prey. Many events in life have a periodic sequence which relies on the value of the other party, and is independent of time. One of these is love between two people, the “Romeo and Juliet” model. This model is between two people, where Romeo’s love increases with Juliet’s, but Juliet’s decreases as Romeo’s increases. This will produce a periodic graph similar to the Predator prey model. This is also seen in models of Guerilla warfare, where the strength does not solely rely on the sheer number of either force because they can hide in buildings, but the casualties of each side are proportional to both their own numbers and the opponent’s numbers. So if one side has less men, they will also die less, because they can hide more easily and cannot all be shot at once, like open warfare.

Although an equation such as the Lotka-Volterra model can roughly estimate the populations of animals and show the general cyclical behavior of the ecosystem, it does not take into account many minute factors which can greatly impact an organism’s population. These factors all pertain to the traits and behaviors of individual organisms and species, as each organism is its own entity which acts on its own accord. When examining actual data of populations in ecosystem, you can draw trends from the minutest of scales, such as months, to the greatest amounts of time, such as millions of years. As you expand your analysis of ecosystems onto a longer, broader scale, trends can become more esoteric; as there are a massive number of species which interact with each other, and the cause of extinctions or population inflations can seem uncaused. However, one thing is certain: all ecosystems follow periodic trends.

4.0 Procedural Overview

4.1 Algorithm

Our Algorithm uses three different actors that do all the actions. In the beginning of our program a function will fill arrays with these actors based on a seed. For the purpose of the program, these arrays can be thought of the terrain in which the actors live. After our ecosystem is initialized, a function calls on each actor to conduct an action for the current segment of time. We have decided to make each time one month and provided a basis for the values in our actors (see the graphic on page eight for a detailed description of the functions).

The simplest actor in our algorithm is the producer, otherwise known as the plant. The plant is simply is an entity that grows every month, until it is fully "alive" after several months. Partially grown plants cannot be eaten. Each square in the plant array has a 50% chance to contain an alive plant in the control run. This makes a heavy amount of plants, which is crucial to the model because the Lotka-Volterra models were based with the prey having a nearly limitless supply of food. Every time a plant is eaten and there is a consumer in the square, the plant is set to dead and has a "maturity" of 0. Once the consumer leaves the square the plant starts to grow again.

Both the primary consumer and the predator have similar rules in our algorithm. At the beginning of each "month", there is a function which checks the status of each animal in the array. These functions also include the breed and birth functions. Our animals have genders, and if the animal is a female then it checks if there is a male in its range to breed with. If it does, then it becomes a "mother". Once the animal is a mother, it has a "nest" value that increases once every time step. (cont. on page nine)

Initialize

- This initializes the plant, then the bigfoot, and finally the dinosaur array
- For each species, If a random number between 1-100 is less than the "Initial value," an object of that type is placed there

Life check

- Each bigfoot and dinosaur go through a series of chencks at the beginning of the timestep
- First it will check to see if the animal is a mother, and if it can give birth
- If not , it will try to breed if it is a female
- After that it will check its age, and it will die if it is older than the maximum
- Finally, it will check to see how hungry it is, and it will die if it hasn't eaten in a certain period of time

Bigmeal

- This is how a bigfoot eats
- If there is a plant in the range it will place that square in a possible square
- The bigfoot will randomly choose one of the squares with a plant on it
- This takes up its movement, preventing it from eating more than once

Plant regen

- Every plant will grow in maturity if there is not a bigfoot there
- If the plant reaches a certain maturity, it can now be eaten by a bigfoot

Dinomeal

- Very similar to the bigmeal except for dinosaurs to eat bigfeet
- In addition, the dinosaur will move to a random square next to it if cannot find any bigfeet
- Unlike the plants, bigfeet have a chance to escape from being eaten

The breed function is called once the nest value is equal to the gestation value. This function will create a new animal in a consecutive square that is randomly decided. Then a consumer will check for any living plants without a predator in them. Since a predator only eats consumers, they can be located on the same square as a plant, while a consumer must be on its own square. After the consumers move the plants regenerate, then the predators will eat. Each animal has a large range of 7*7 since they will die so fast if they cannot find food. There are checks to make sure each animal can only move once per time step. Each animal will automatically die if they do not eat or they reach a certain age. It is important to note that there are three separate arrays, and each object does not directly interact with an actor from another array.

Another feature we used in creating and collecting data were input/output files. Altogether we ran over 100 trials with different parameters, so these files allowed us to quickly run our program without recompiling it every time. It also placed the data in a spreadsheet where it could be graphed and organized.

4.2 Visualization

When using a small ten by ten or twelve by twelve grid, we visualized our simulation simply using the command prompt. This proved very helpful when debugging, because we labeled each individual so we could tell if it was moving more than once. We represented each space in the grid by a horizontal bar to separate elements in the row, and nothing to separate elements in a column. A plant is a P, a Bigfoot is a B, and a dinosaur is a D. A dinosaur and a plant located on the same square is represented by a H. However, these dimensions are too small to give good results due to the behavior of the predators, and the results we gathered were by using a grid two hundred by two hundred. See figure 5.01 for an example of this visualization. Note, that on the final code, we have omitted these functions for greater efficiency.

5.0 Results

In our program, there are two global variables which we didn't change while collecting data. The first is the seed value; we used zero for this, which provides a new, random seed upon every run. Also, we didn't change the run length of 3000 iterations. In this model, every iteration roughly represents a month. However, our program has 11 global variables which we altered in order to see how they would affect the ecosystem:

- Initial Values (Plant, Primary Consumer, Predator)

    These values help determine the initial percentage of the ecosystem's space that is covered by either plants, primary consumers, or predators.

- Age (Plant, Primary Consumer, Predator)

    In the case of primary consumers and predators, these values place a limit on how many iterations they can survive before they are forced to die. Generally, these entities will die because of starvation or being eaten, but this helps ensure that they do not remain in the ecosystem for an unrealistic amount of time.

    In the case of plants, the age variable determines how many iterations it takes for a forested square to completely regrow.

- Hunger (Primary Consumer, Predator)

    The hunger variable determines how many iterations a primary consumer or predator can survive without food before starving. This value cannot be changed a significant amount, because in general, animals cannot survive for more than a month without eating.

- Gestation periods (Primary Consumer, Predator)

    Gestation periods determine how many iterations a "pregnant" primary consumer or predator will take until they birth a single offspring.

- Defense (Primary Consumer)

    The defense variable represents the percent chance of a primary consumer surviving an attack by a predator.

5.1 Control Model

Parameters:

Our control model used parameters that we thought were representative of a realistic ecosystem in its simplest form:

- Seed Value: 0
- Initial Value Plants: 50%
- Initial Value Bigfeet: 25%
- Initial Value Dinosaurs: 2%
- Plant Age: 5 Iterations (5 months)
- Primary Consumer Age: 84 Iterations (7 years)
- Predator Age: 168 Iterations (14 Years)
- Primary Consumer Hunger: 1 Iteration (1 month)
- Predator Hunger: 1 Iteration (1 month)
- Primary Consumer Gestation: 3 Iterations (3 months)
- Predator Gestation: 6 Iterations (6 months)
- Run length: 3000 Iterations (250 years)
- Defense: 0%

Results:

The control code was ran 50 times, which helped us distinguish some patterns. We averaged the 50 runs, and were able to create several graphs which help show trends in the data(see figures 1.00-1.11 on Appendix 8.1 for graphs of the averages).

With the control code, we are generally able to find a relative equilibrium between the species. As previously stated in the report, a true equilibrium is never established, as the populations of predator, prey, and producer, cyclically increase and decrease. However, after a certain period of time, we find that almost all data points end up in a tight area which we call the “relative equilibrium”. This term means that all of the organisms’ populations vacillate in a very minor fashion, as opposed to the beginning of the ecosystem which sees great change.

Because of the random nature of the program, we can find variation between the different runs. For example, in three of the 50 runs, we saw the extinction of the predator species at 555, 796, and 906 iterations in, respectively. If you consider extinction the existence of only one dinosaur, then the extinctions occur at 475, 655, and 899 iterations. In the first case, it is apparent that the age limit prevented the final dinosaur from living an absurdly long time, as it could find a high concentration of food with no competition. In the second case, it is likely that the final dinosaur became pregnant before the second to last dinosaur died, and gave birth before it died. Its offspring eventually either died of age or starved.

Although the dinosaur populations occasionally die out, in no run did the bigfoot population die out. This is not representative of the real world, and is likely due to the overwhelming forestation.

On a macro scale, once a relative equilibrium is reached, we are able to see very minute changes in the organism's populations (see figures 1.00, 1.02, and 1.04). However, on a micro scale, we are able to see fluctuations in the population of the animals. By examining the averages of iterations 2500-3000, we are able to find relationships between predator, prey, and producers (see figures 1.01, 1.03, and 1.05). Ignoring the smaller troughs and peaks, we can see a rough, parabolic trend within the 500 iterations. For forestation and dinosaurs, we see a relative maxima in population at approximately 2775 months in. With bigfeet, we see a relative minima at the same time. This shows that in ecosystems, the populations of a predator and producer are directly proportional; whereas the populations of a primary consumer are inversely proportional to the previously stated two. When examining the graphs, we can see that even the smallest peaks and troughs are correlated to eachother.

In sections 5.2-5.4, we experimented with the values of several variables, using three runs for each change in a variable. For example, with plant age in 5.2, we used values of three, four, six, and seven. For each of these variables, we ran three trials. This resulted in 12 sets of data overall.

5.2 Variations in Producer

We experimented with the age value of the plants, using values of three, four, six, and seven. Notice that five is omitted since it is the default value of the plant age. In this context, the plant age value refers to the amount of months it takes for a plant to become fully grown. Using three, four, and five (the default) values rarely provides dinosaur extinctions. However, using a value of six or seven will results in dinosaur extinctions nearly 100% of the time (see figures 2.00 and 2.01). This means that the threshold of plant age which will result in 50% of extinctions within 3000 iterations is in between five and six. With a value of six, we see extinction occur between 517 and 582 iterations. With a value of seven, we see extinction occur between 208 and 259 iterations. This is clearly caused by the direct correlation of dinosaur population to forestation. If it takes longer for plants to fully develop and help stimulate the growth of the bigfoot population, it affects the dinosaur population negatively.

We also changed the initial values of the plants, which determined how much of the ecosystem was originally forested. This caused negligible change; the only noticeable effect was that it took longer for the ecosystem to reach a relative equilibrium.

5.3 Variations in Primary Consumer

With the primary consumer, we changed four different variables. These were age, initial value, gestation, and defense chance. With age, we saw very little to no change in the overall ecosystem. However, we did notice that, in general, an increased age limit did result in a slightly more stable ecosystem. This means that the relative maxima and minima once the system reached a relative equilibrium were closer. The bigfoot population, as we previously discovered, is always stable; this is even present after the dinosaur's population has gone extinct. This is likely due to the abundance of producer agents. Changing the initial values also caused very little change, as only one of the results saw a dinosaur extinction—this was likely an outlier.

Changes in the gestation period of the bigfeet, however, saw a dramatic change in the dinosaur population (see figures 3.00-3.03). By increasing the gestation period of the bigfoot, the ecosystem saw a rapid decrease in the dinosaur population. When the gestation period was four, one third of the ecosystems saw a dinosaur extinction. With a gestation period of six, all ecosystems saw the extinction of dinosaurs within 500 iterations. The graphs of gestation periods of eight and ten are almost indistinguishably similar; in fact, a gestation period of eight iterations shows a quicker extinction than a gestation period of ten iterations. This can be associated to two causes: one, because of the random nature of the program; and two, because, after increasing the gestation period past eight iterations, it is impossible for the dinosaur population to survive. The dinosaur population, with a bigfoot gestation period of eight or higher, survives until approximately 24 iterations due to a lack of food in the system to sustain themselves.

Altering the value of the defense chance also proved to cause great change in the ecosystem. Even raising the chance of a bigfoot to defend themselves by 10% saw the extinction of dinosaurs between 134 and 217 iterations. Further increasing the chance for a bigfoot to defend itself caused an even earlier extinction of dinosaurs (see figures 3.04-3.06). It is very interesting how even increasing the defense chance a minute amount can immensely effect an ecosystem. This is because every interaction between every species is related; if one dinosaur is denied food, and dies, it reduces the probability that another dinosaur of the opposite sex could breed with it.

5.4 Variations in Predator

To investigate the dinosaur population, we changed three parameters: Gestation period, starting populations, and maximum age. Each time a parameter was changed, three runs were conducted, and the results for each one were the average of all runs with that parameter. For the most part, we only graphed the population of dinosaurs relative to plants and bigfeet because there was not an observable correlation between the dinosaur population and time.

Most of our results followed similar patterns even with different parameters. The first few months were understandably the most volatile, and often started with high numbers of dinosaurs, and they quickly died as their prey disappeared. The system started out like the Lotka-Volterra equations would suggest, but did not have the same periodic cycle because it spiraled around a point (the center). An unexpected trend that existed was the similarity between the dinosaur vs. plant and dinosaur vs. bigfoot graphs (see figures 4.00 and 4.01). The graph with the plants as the x-axis was essentially the graph of the bigfoot reflected with respect to the y-axis and appropriately scaled. In fact, the plant graph followed the trend more closely than the bigfoot graph. This means that we can accurately estimate populations of the bigfoot species at a point in time if we have the plant graph.

Almost all of the graphs had one smaller loop that sprung out straight to the left of the center. This is the same shape as demonstrated by the Lotka-volterra equations. The dinosaurs reach a population where they will quickly eat a lot of bigfeet without a decline in their own population. Then comes a point when the dinosaurs are impacted by the lower bigfoot population, and drop nearly straight down. The bigfoot population will steadily recover without changing the dinosaur population. Once the bigfoot population recovers to what is was before, the dinosaurs will slowly grow and the cycle will happen again. This cycle is constantly happening, but is only noticeable once or twice because after that it will happen on a smaller scale and closer to the center so all the points are clustered densely together and each period of the cycle is indistinguishable from the next.

Changing the gestation period resulted in small but important differences on the ecosystem. Our control run had a gestation value of 6, and the maximum and minimum values tested were 4 and 8. When the value was 4, the dinosaurs had a relatively straightforward decline, but had more fluctuation in their populations. This means the area around the center was more spread out than the control. The longer gestation period resulted in less average dinosaurs, and therefore, more average bigfeet. As the gestation period grew longer, the beginning of the run became more unpredictable and chaotic, but the area around the center became more concentrated. This happens because the dinosaurs are unable to give birth in the first few months because of the longer gestation period, so the beginning is more unpredictable. After the dinosaurs give birth, because there are fewer population spikes that result from shorter gestation periods. The center hovered around 250 dinosaurs, 5,250 bigfeet, and 13,000 plants when the value was 8. However, when the value is 4 the center changes to around 300 dinosaurs, 4,750 bigfeet, and 15,000 plants. Changing the initial value produced the greatest effects on the ecosystem. The initial value of one produced the graph that was smoothest and appeared most like the Lotka graph. This is because there were fewer dinosaurs to begin with, so the system experienced fewer shocks in the beginning.

When the initial value was increased to two, the populations became more unpredictable and jumped around more in the beginning months. However, when the initial value changed to three, a sharp contrast occurred (see figures 4.02 and 4.03). There were so many dinosaurs at the beginning that many died of hunger and could not recover enough to survive. The dinosaurs died around month 1200, but they stayed under 20 individuals since the 900^{th} month. Because we used a random seed, not every run will result in the dinosaur's extinction, but it is a stark difference from the initial value of two, when there is almost a 0% chance of extinction. When the initial value was further increased to four the dinosaurs died even more quickly, in only about 200 months.

The age component affected the overall outcome the least of the variables we changed. This happened for two reasons. Firstly, since they will die if they do not eat for one month, they will mostly all die from hunger rather than age. Secondly, while we doubled the gestation period from four to eight and quadrupled the initial values from one to four, the increase from 144 to 192 is only 33%. Essentially we are deviating less from our control run with our age tests.

6.0 Conclusions

Establishing a stable ecosystem using our algorithm can be a complicated task. Even the slightest change in the parameters for our ecosystem saw drastic change. It was easiest to see the effects of a parametric variation by examining the population of the dinosaur class. This is because plants do not have any requirements to grow, other than an empty space in the ecosystem. Also, plants do not have an age limit; therefore, plants will never go extinct in our ecosystem. Because of this abundance of food, bigfeet populations will fluctuate, but not as violently as that of the dinosaur population. Only in our earlier algorithms did bigfeet go extinct. However, dinosaur populations often go extinct, due to a slight drop in the bigfoot population. Food becomes scarce enough to the point where the dinosaur population cannot sustain itself (see figures 2.02 and 2.03). Although the bigfeet are able to sustain themselves, they do see a dip in population. This causes the dinosaur population to dramatically dip, and eventually reach extinction. The extinction of this predator class does have a great effect on the bigfoot population. In figure 2.03, you can see that near the extinction of the dinosaur class, the bigfoot population sharply increases and decreases. This can be attributed to the removal of a population check—in a way, this is very similar to the Italian fish populations, which were greatly destabilized upon the removal of a predator.

Although our algorithm is not perfect, it does display the volatility of the ecosystems we find on earth. The effects of human behaviors such as overfishing, urbanization, and pollution can greatly destabilize a local ecosystem. As Volterra was able to see, ecosystems may also adapt to human interaction and become stable because of activities such as hunting or fishing. A sudden removal of this interaction can cause the destabilization of the ecosystem, which is the reason that the fish population In Italy after WWI was greatly reduced. Even our simple algorithm shows just how complicated ecosystems can become due to the amount of interactions between species. Being able to forecast how different human interactions will affect an ecosystem will become of increasing importance in the 21st century.

7.0 Future Work

Due to the open ended nature of our code, there are many areas for improvement that we have recognized, and taken an interest in. However, we have not committed to developing any of the proposed code which follows.

With this model, there is a tremendous amount of room for expansion. Originally, we wanted to see how a dominant trait would emerge in an ecosystem due to the surrounding environment and other animal populations. For the sake of simplicity, we omitted both traits and environment from the final model in favor of examining the changes that occur in animal populations. We were also interested in seeing the effects of events, such as deforestation, on an animal population.

We also saw that the movement patterns of animals had a great effect upon the stability of the ecosystem. We programmed the predators and primary consumers to value tiles which had food on them higher than tiles which were void of food. However, for a period of time, we only allowed the animals to see within a square of their tile. During this time period, we saw that dinosaurs often went extinct very early in the life of the ecosystem; within 100 iterations. However, after increasing the range of their ability to find food, we saw that the organisms found a relative equilibrium much more often. These changes were included in our control code. However, behaviors in movement within animals are highly complicated, and unique often times. It would be very interesting to implement these behaviors and see the effects on the ecosystems.

Our model is also very simple, in that we only use three species. In real ecosystems, thousands of species from all kingdoms are present. While modeling individual bacterium would be near impossible, decomposition on a broad scale could be implemented to help check the growth of plants. Including different kinds of animals in our ecosystem, such as omnivorous consumers, could help increase the realism of the model. Adding several different species would be possible, but by doing this, we would likely want to increase the space of the ecosystem. Since our PCs were not able to run an array larger than 210 by 210, this would be an excellent task for a supercomputer.

8.0 Appendix

8.1 Appendix A: Average Control Run Graphs (Figures 1.00-1.11)

Figure 1.00

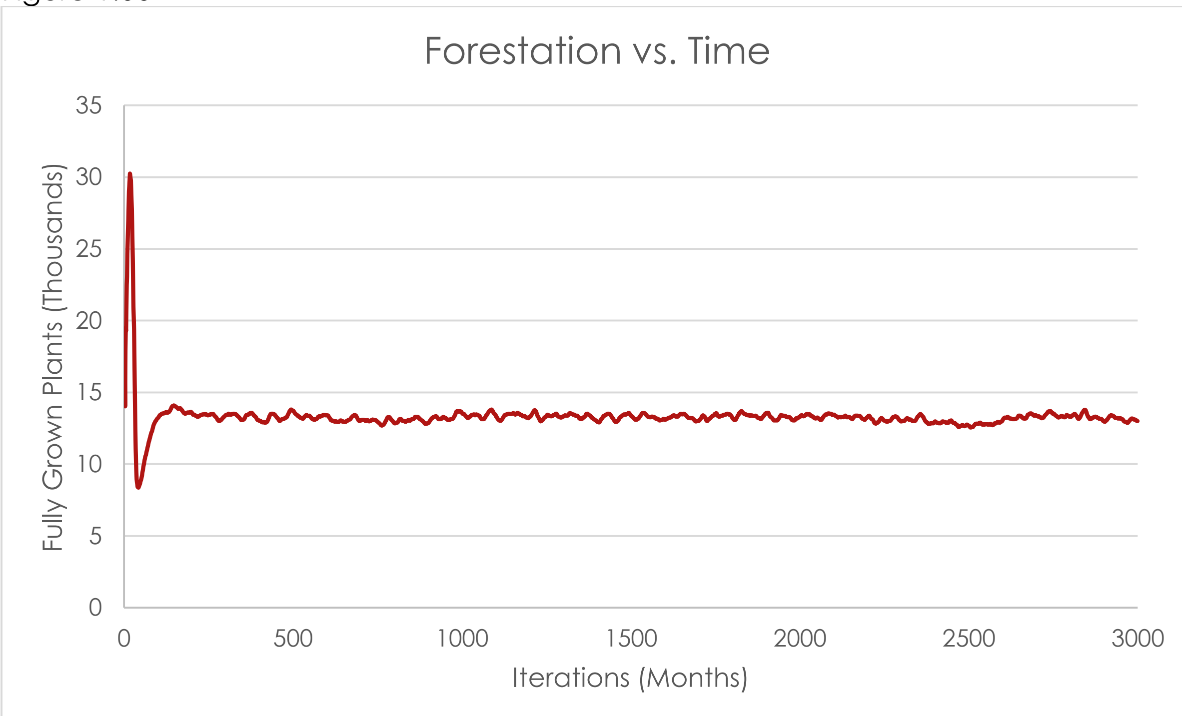
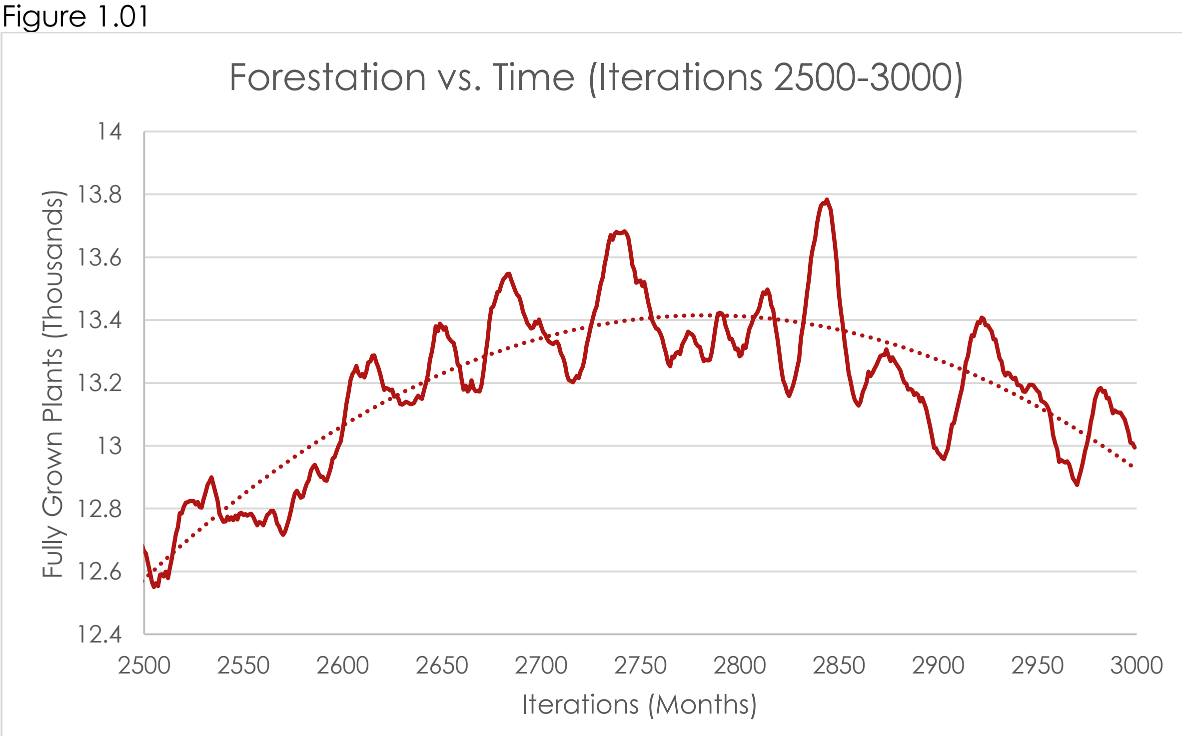


Figure 1.01



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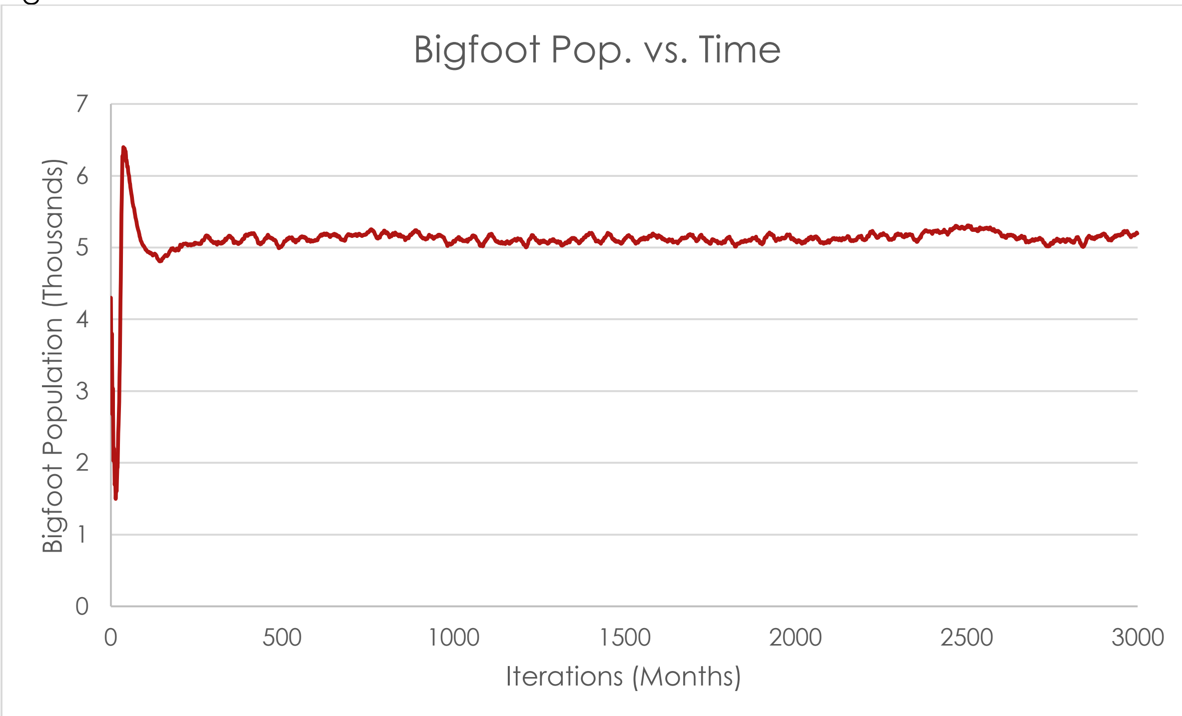

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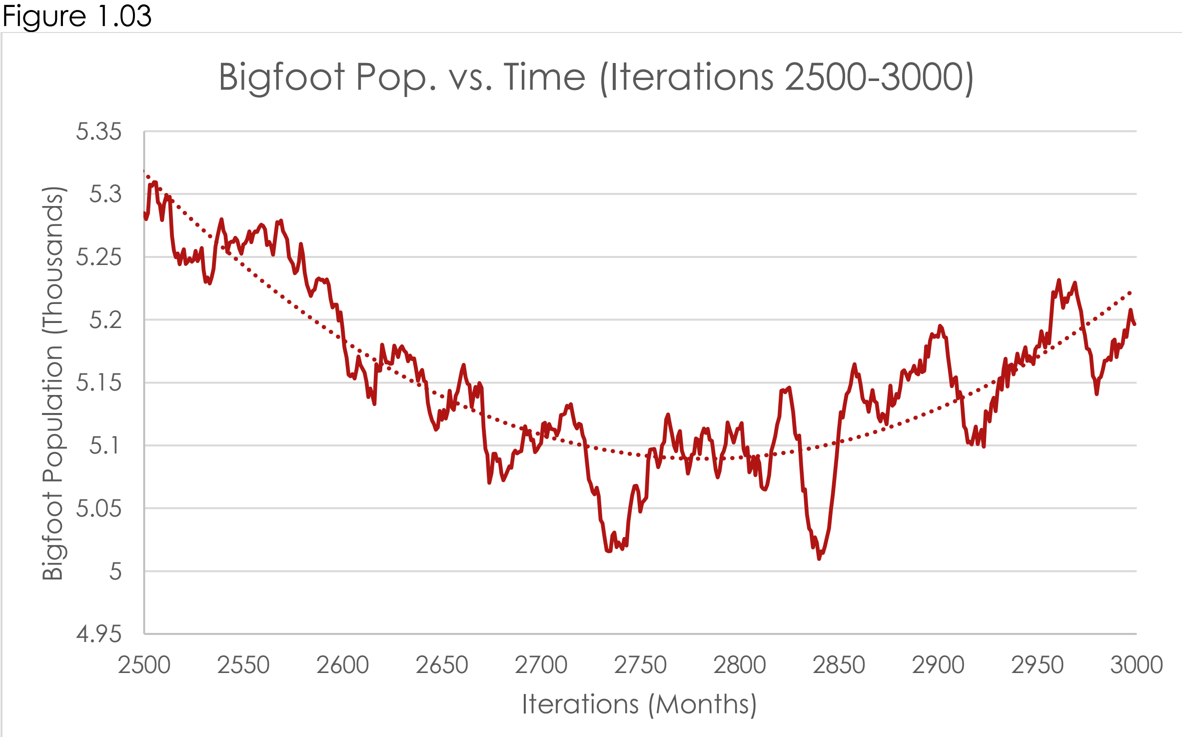

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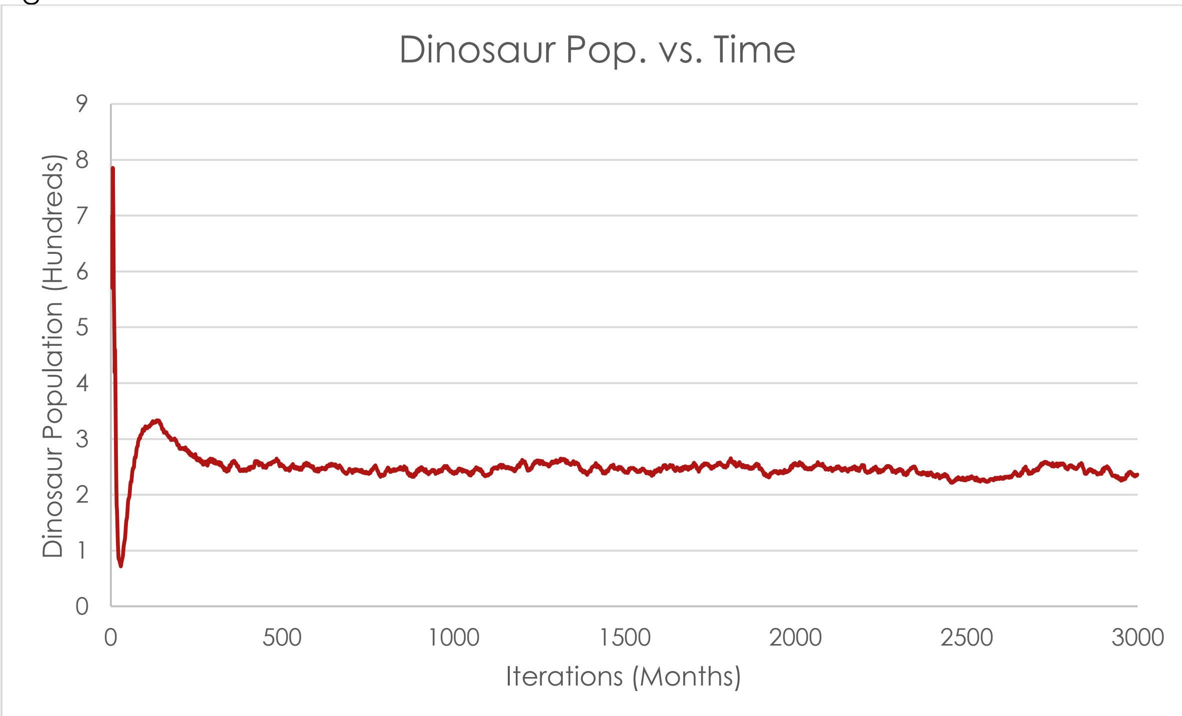

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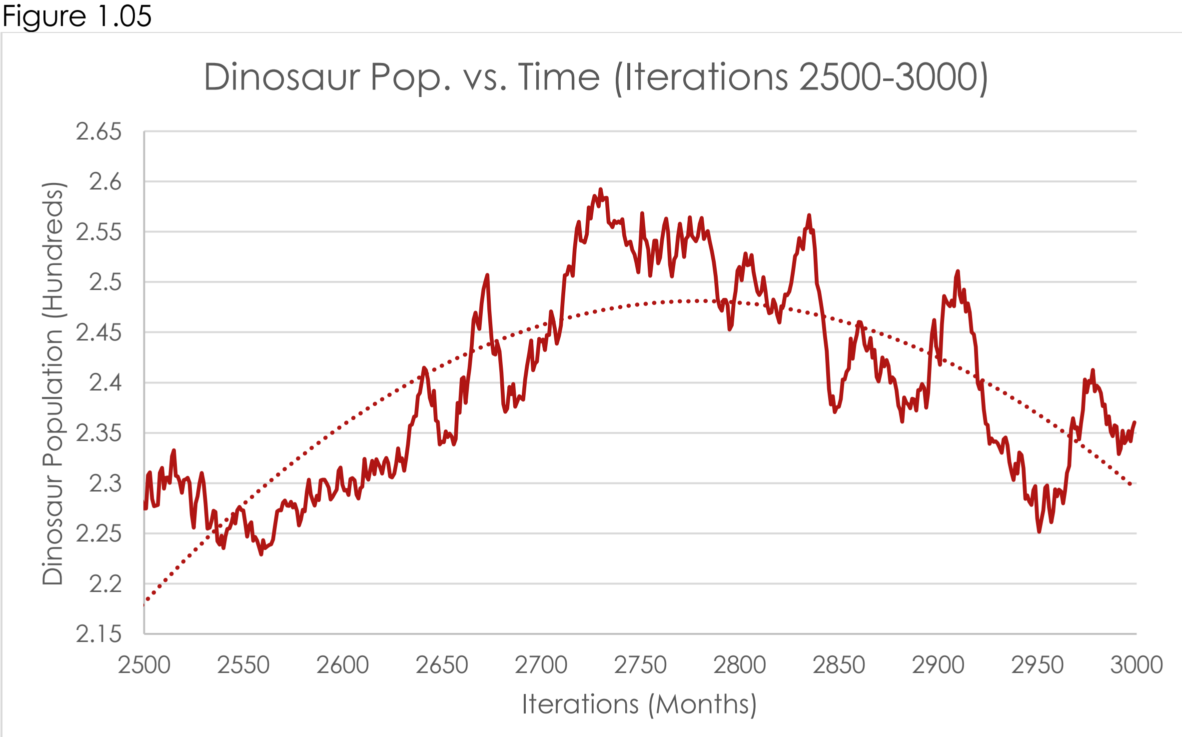

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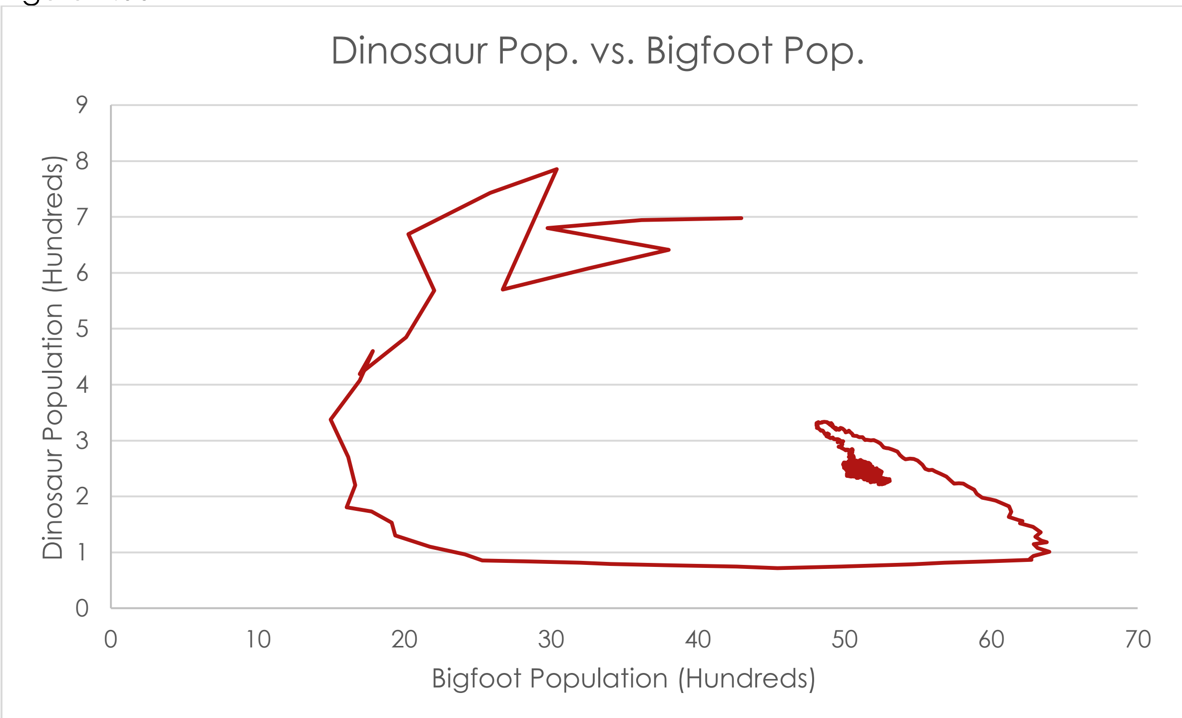

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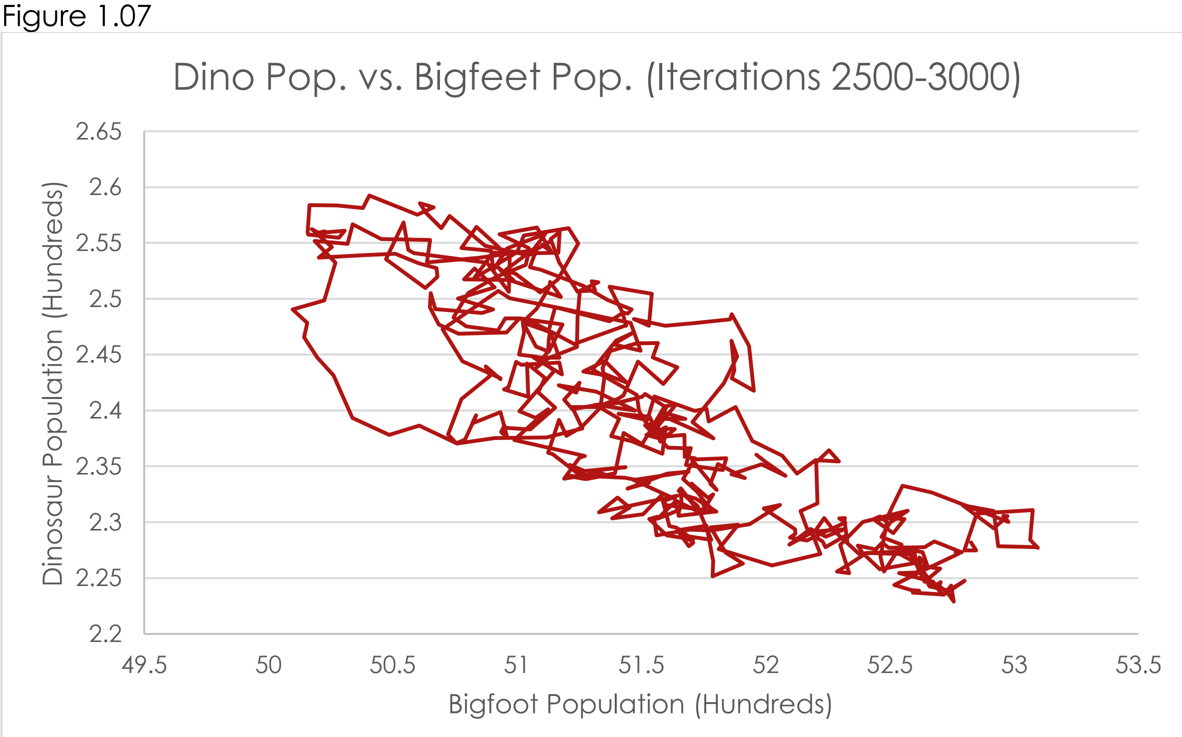

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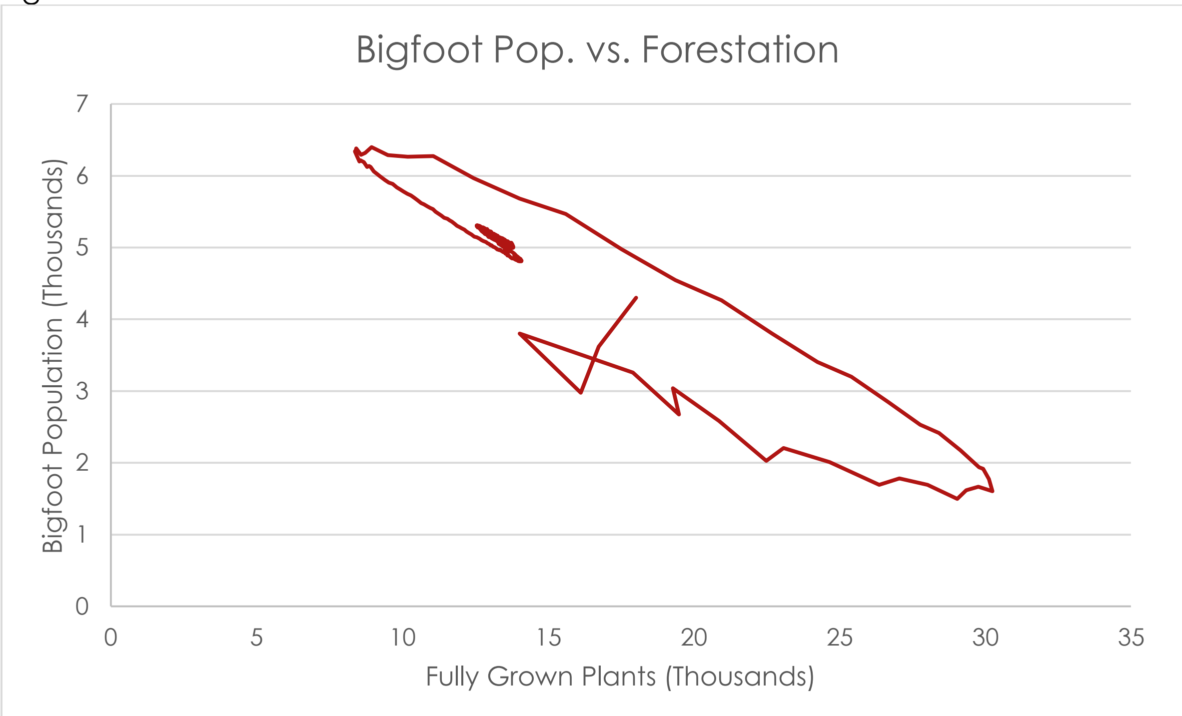

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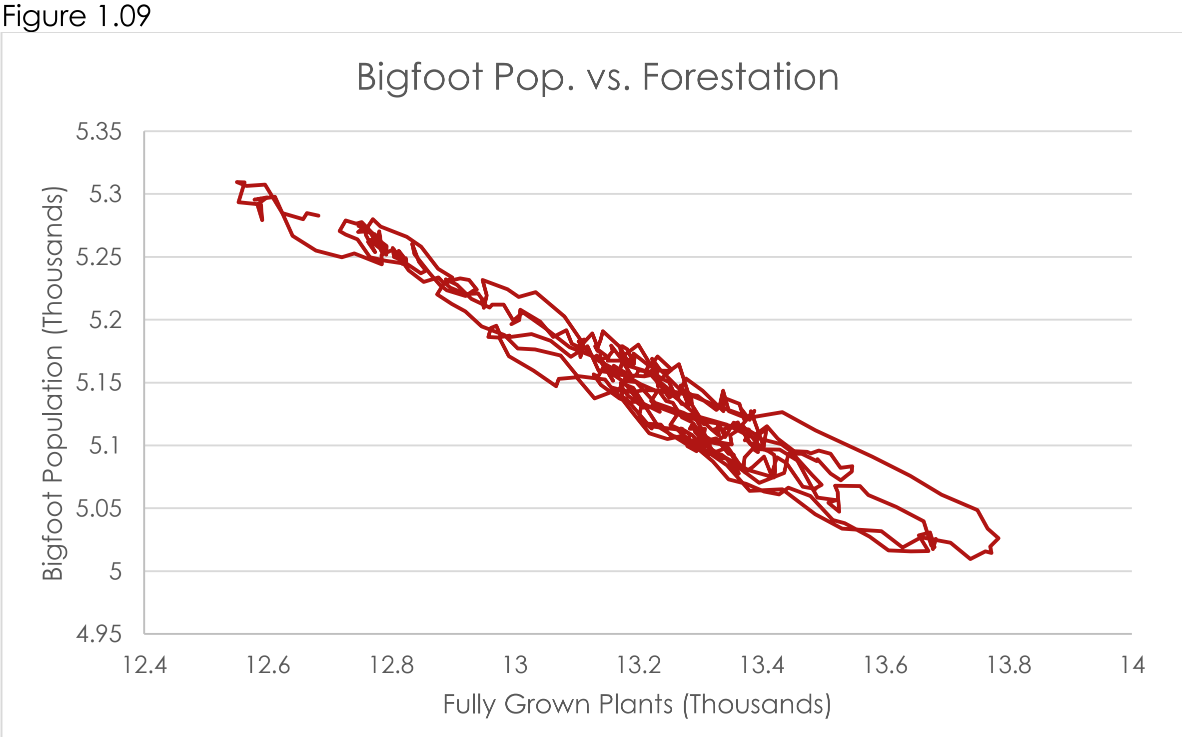


Figure 1.10

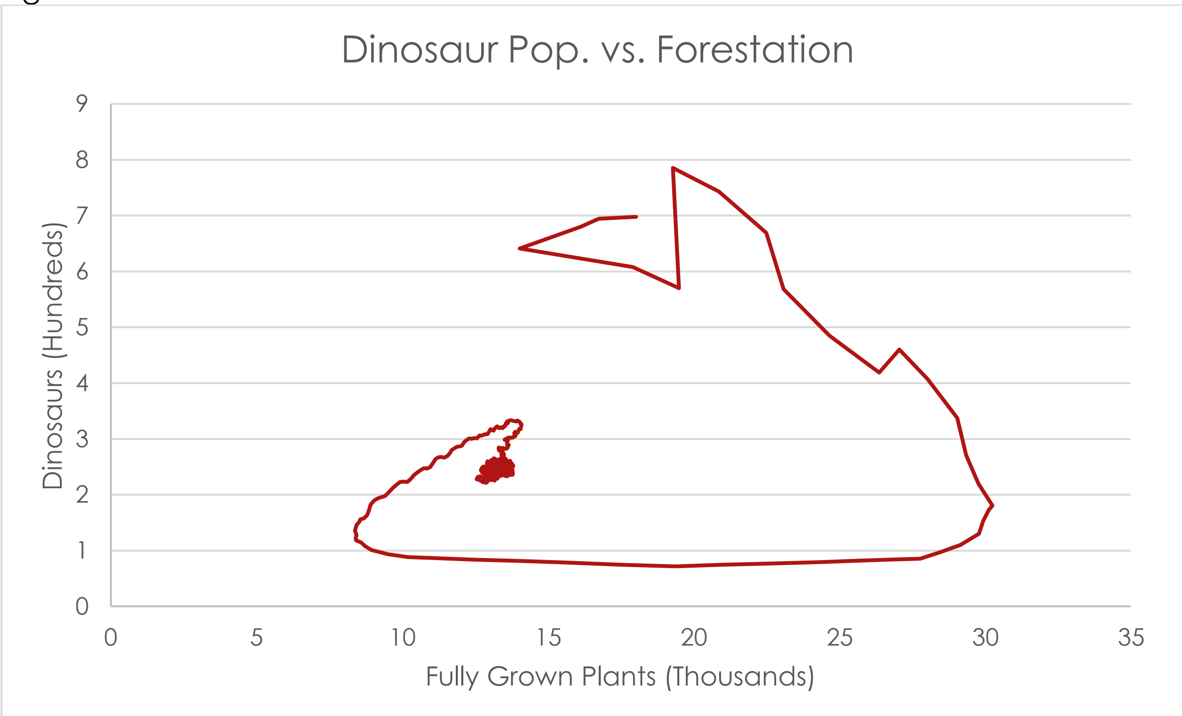
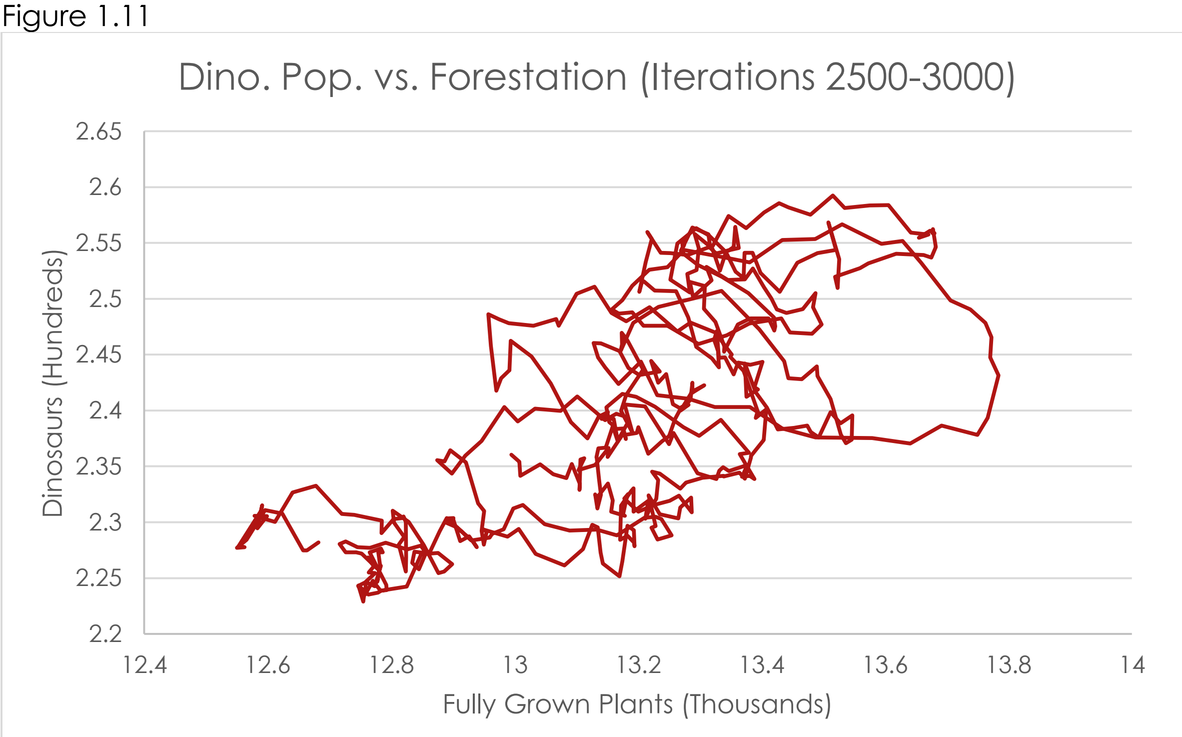
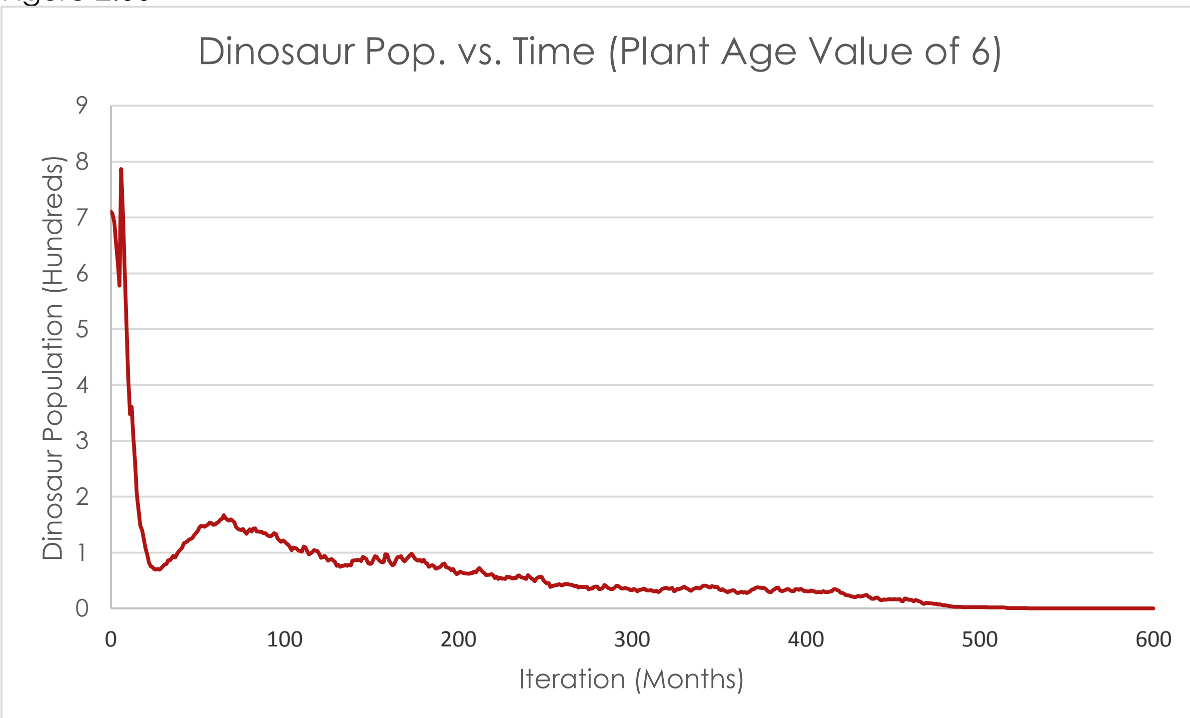


Figure 1.11



8.2 Appendix B: Plant Variable Changes (Figures 2.00-2.03)

Figure 2.00



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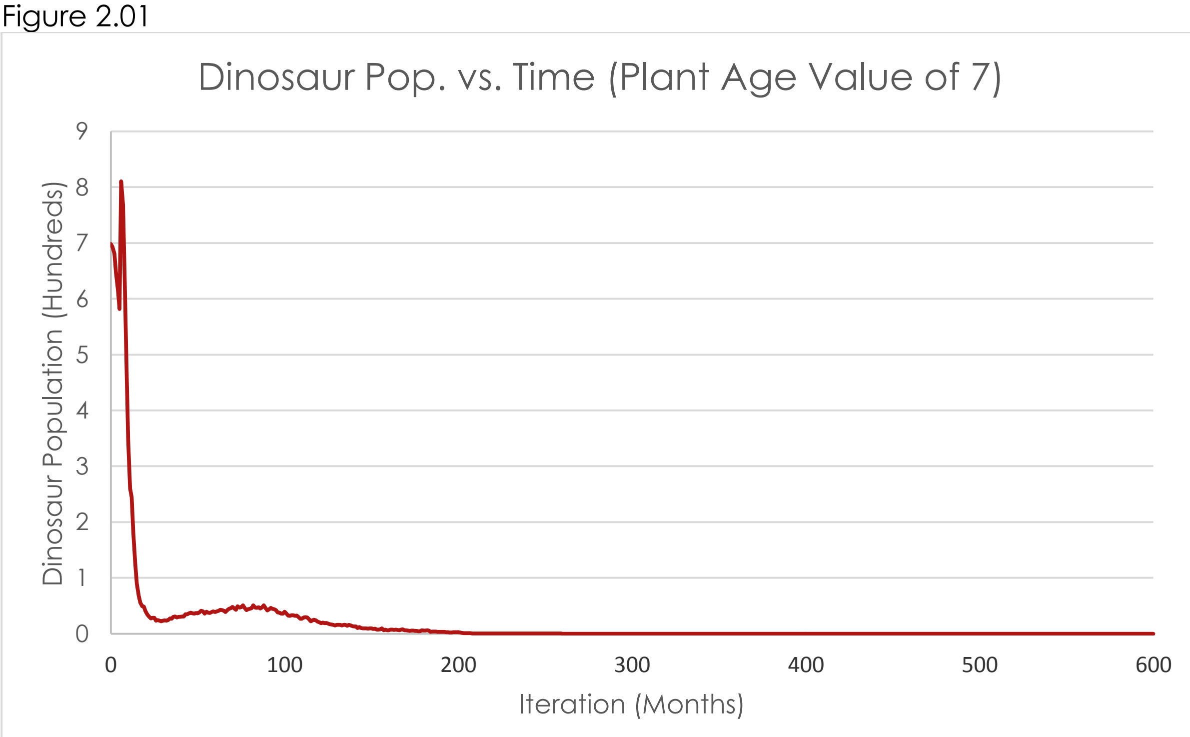

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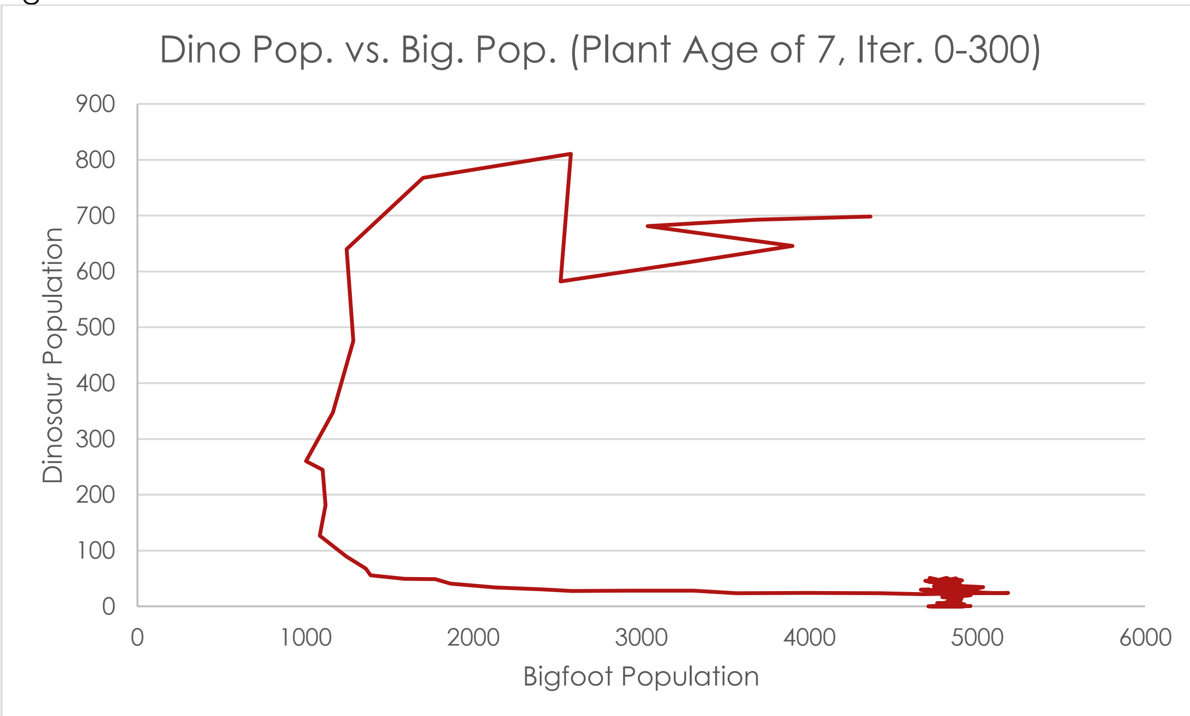
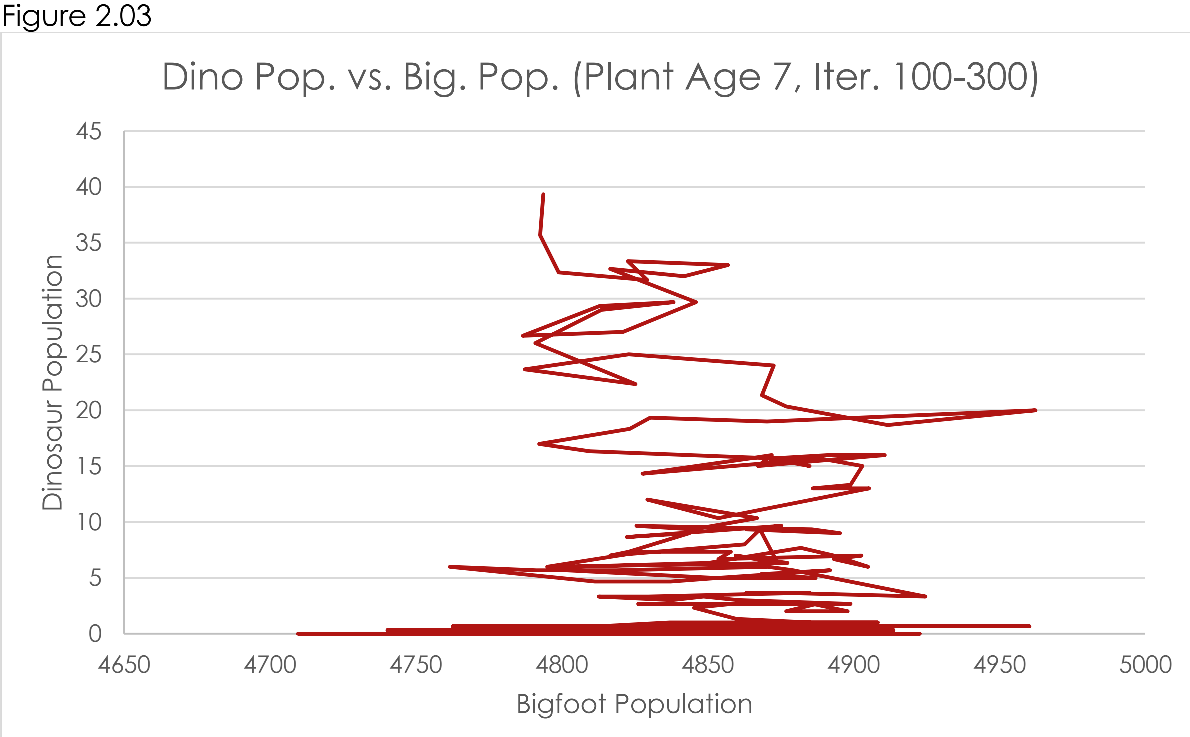


Figure 2.03



8.3 Appendix C: Primary Consumer Variable Changes (Figures 3.00-3.06)

Figure 3.00

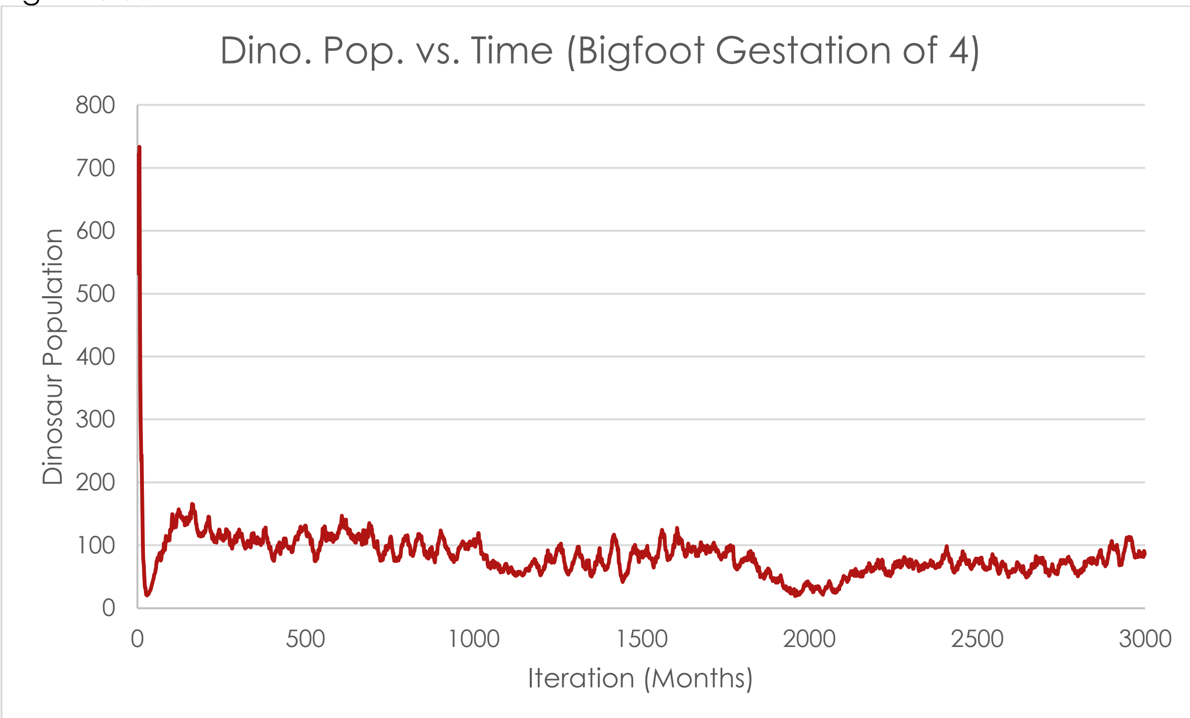


Figure 3.01

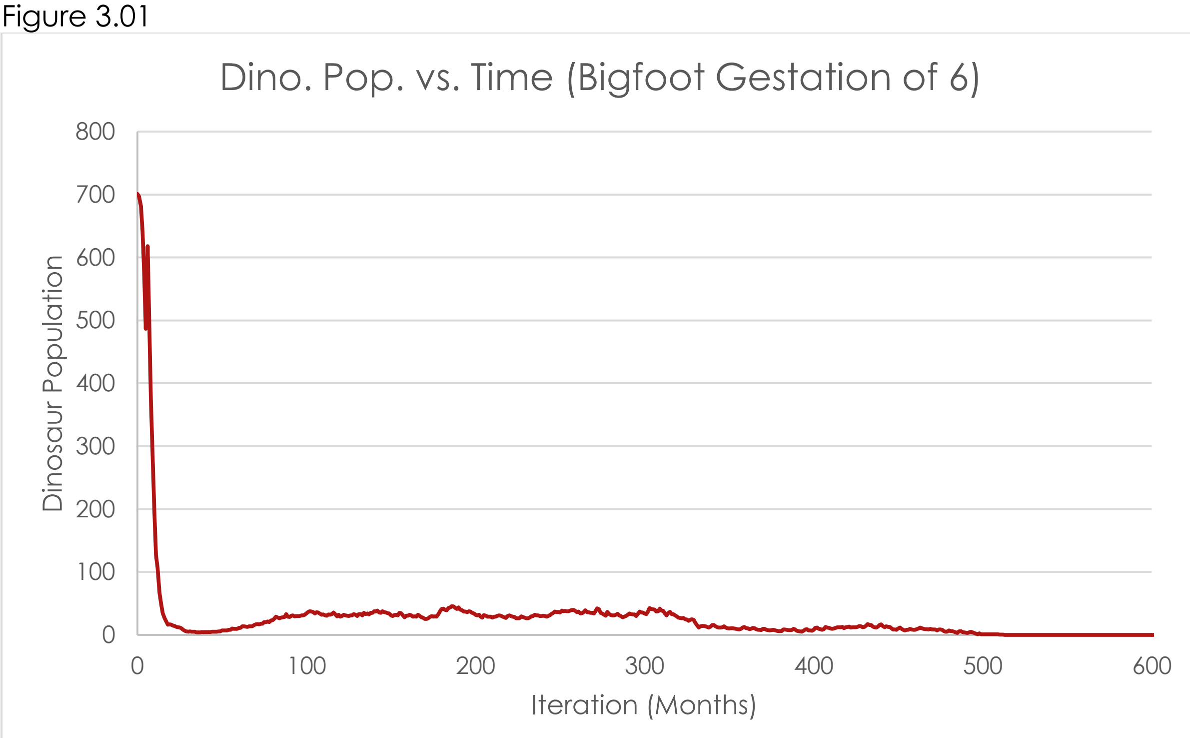


Figure 3.02

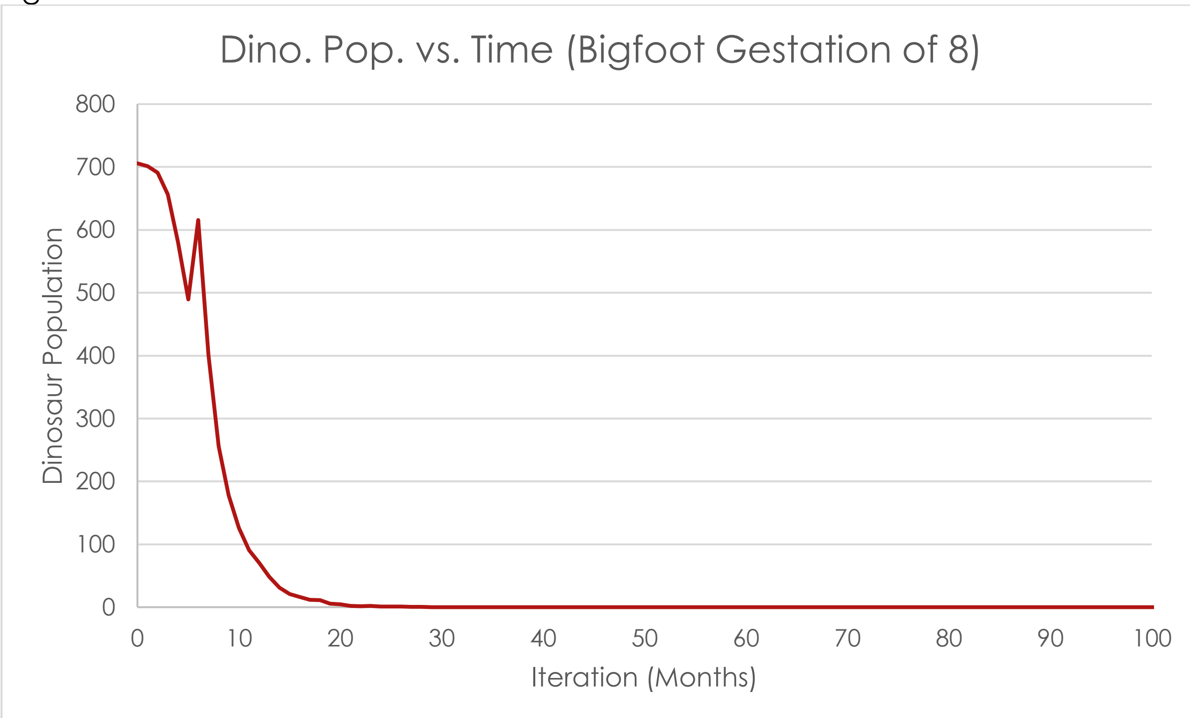


Figure 3.03

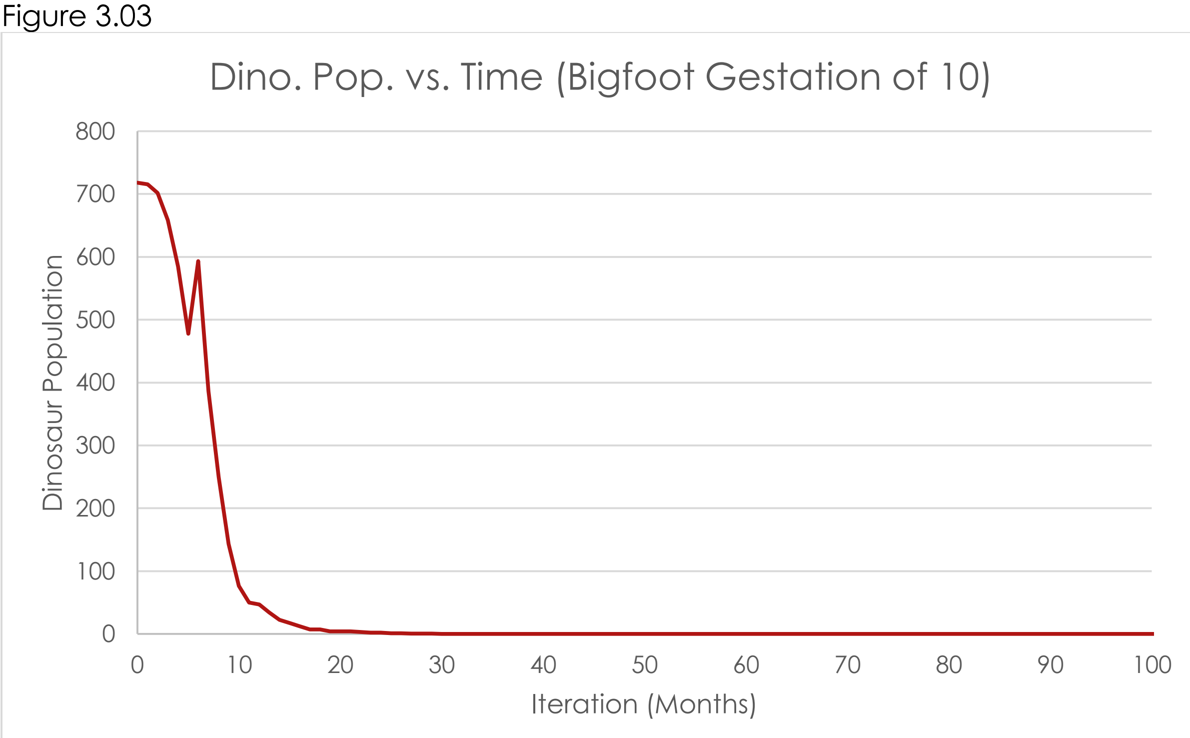


Figure 3.04

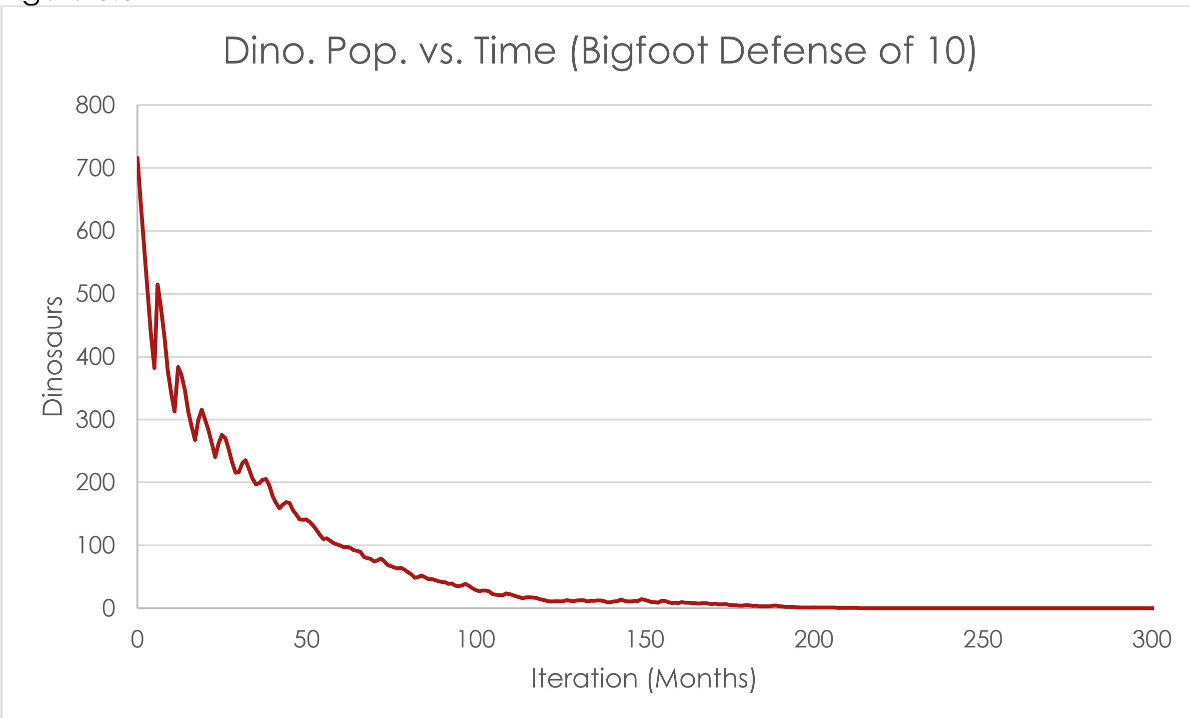


Figure 3.05

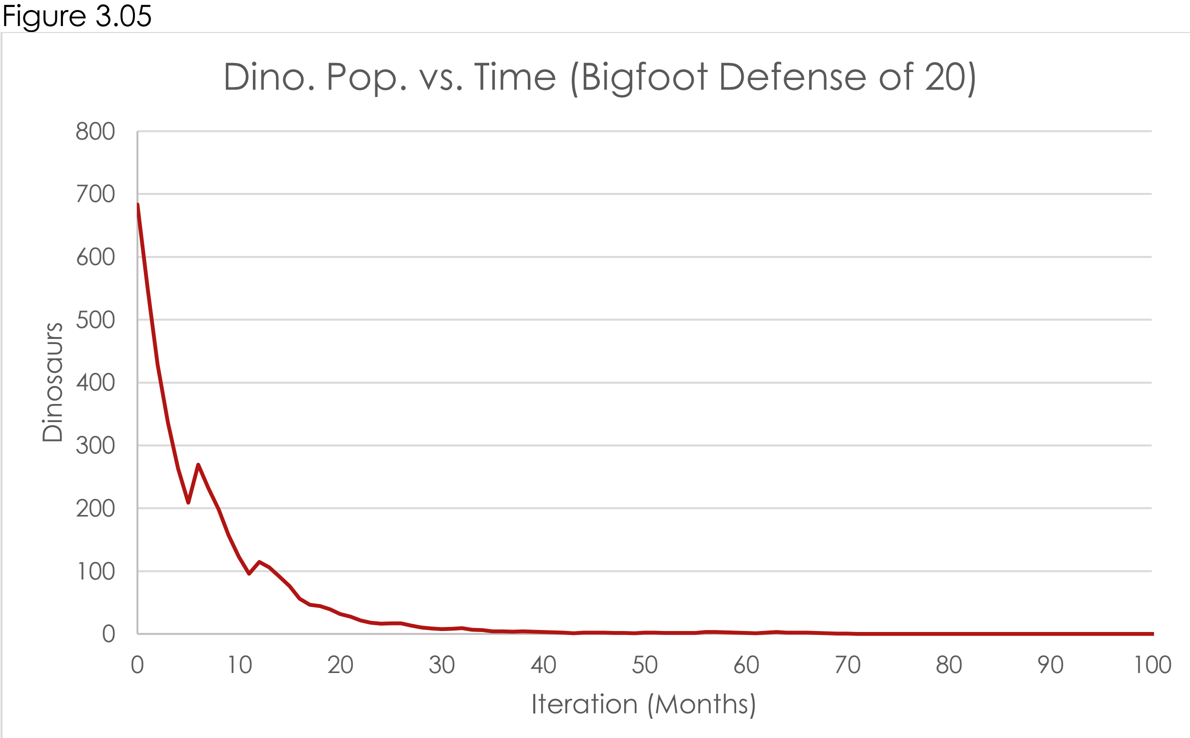
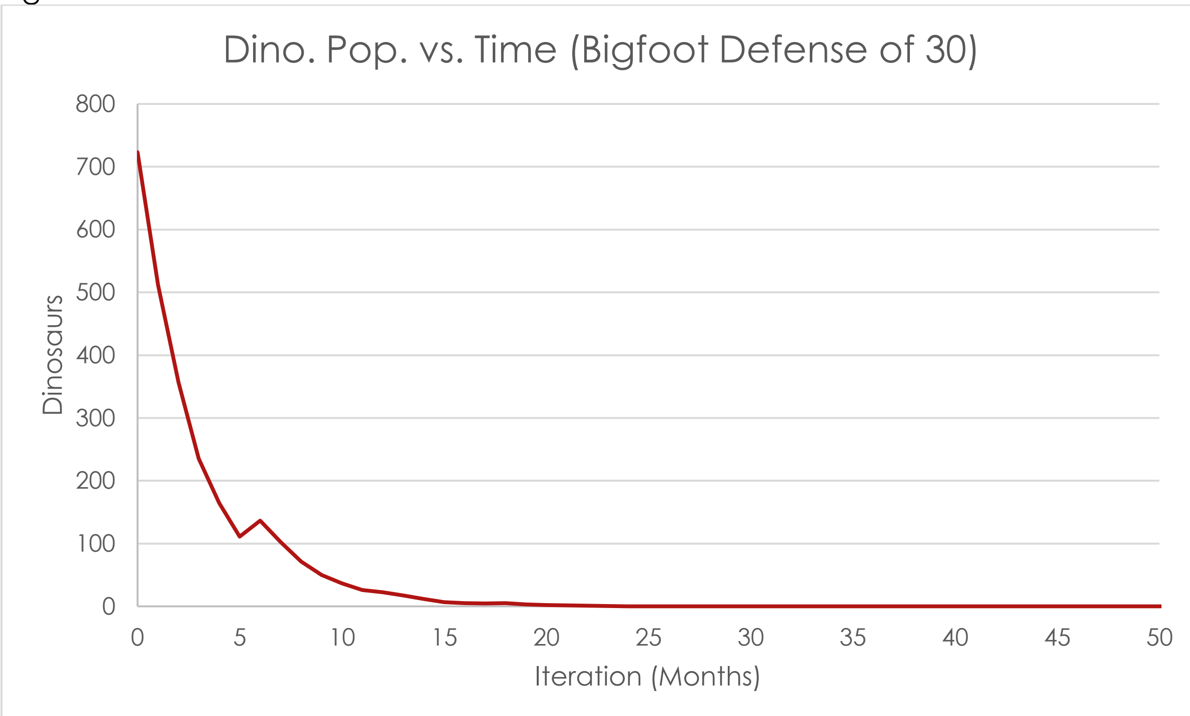


Figure 3.06



8.4 Appendix D: Predator Variable Changes (Figures 4.00-4.01)

Figure 4.00

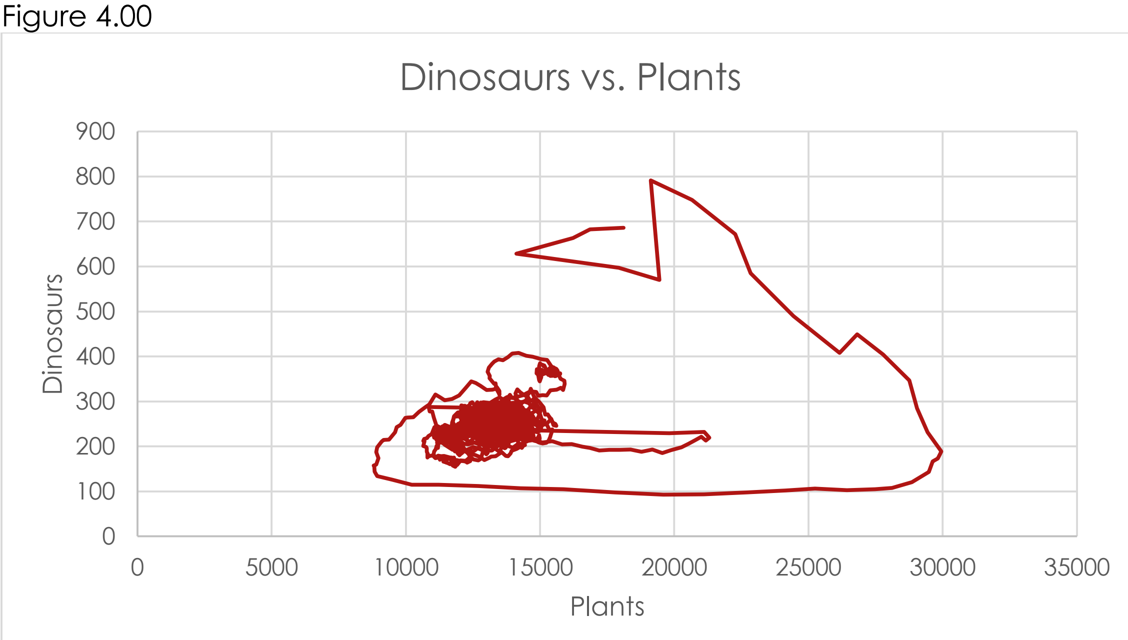


Figure 4.01

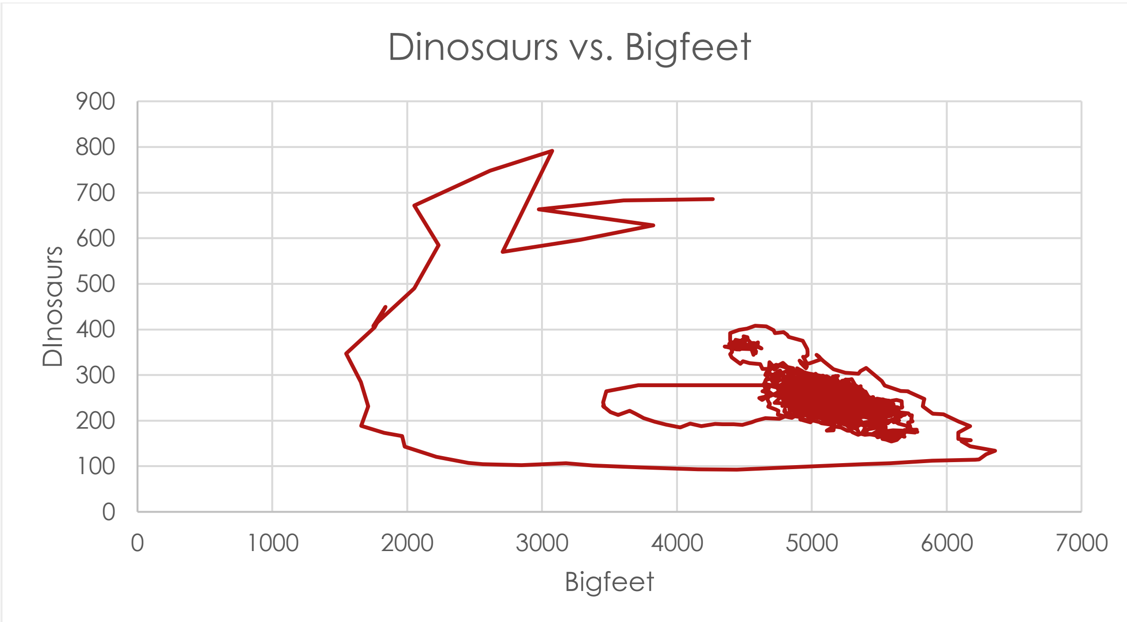


Figure 4.02

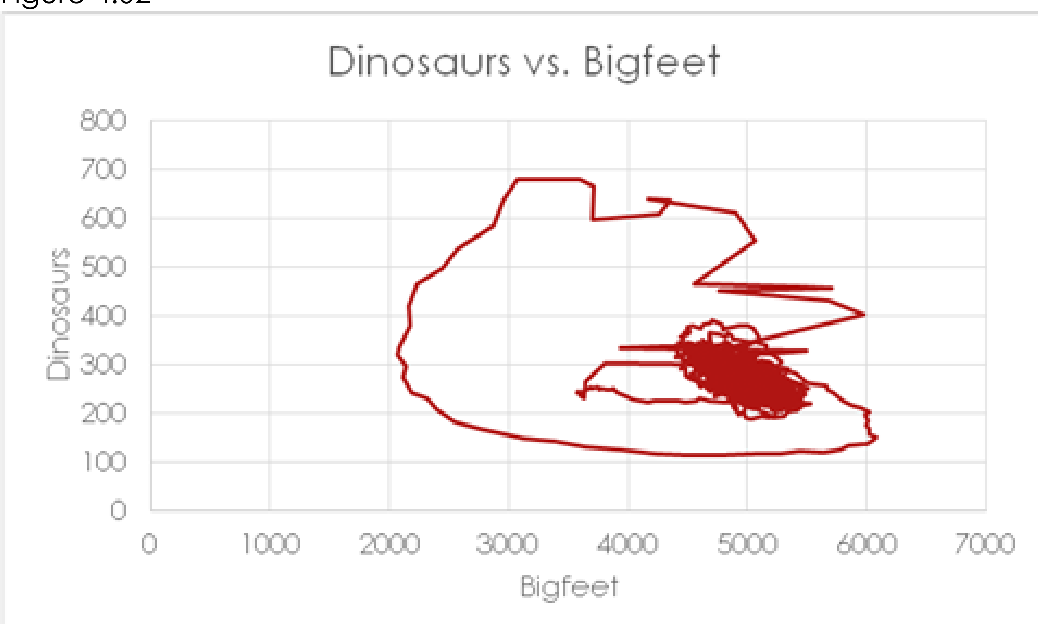


Figure 4.03

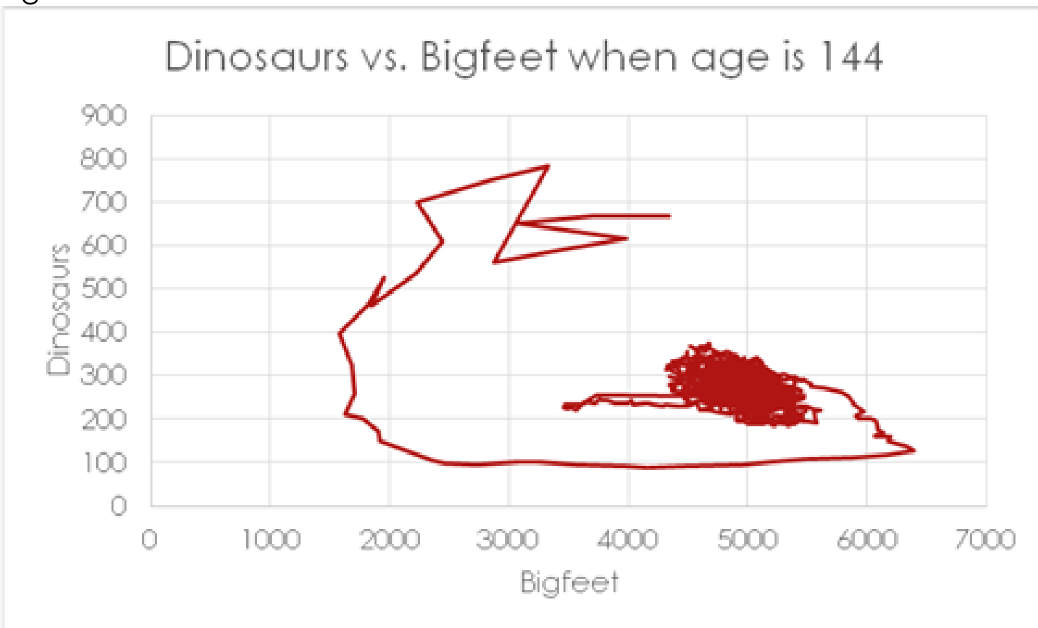
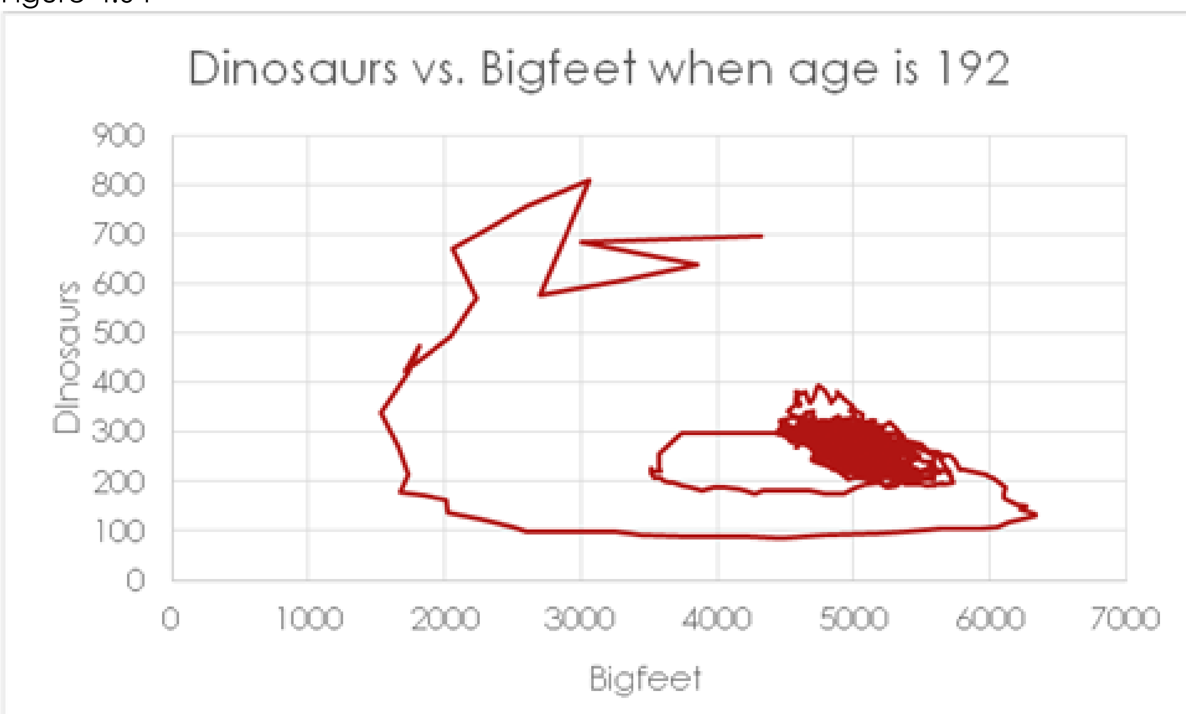


Figure 4.04



8.5 Appendix E: Miscellaneous Figures

Figure 5.00

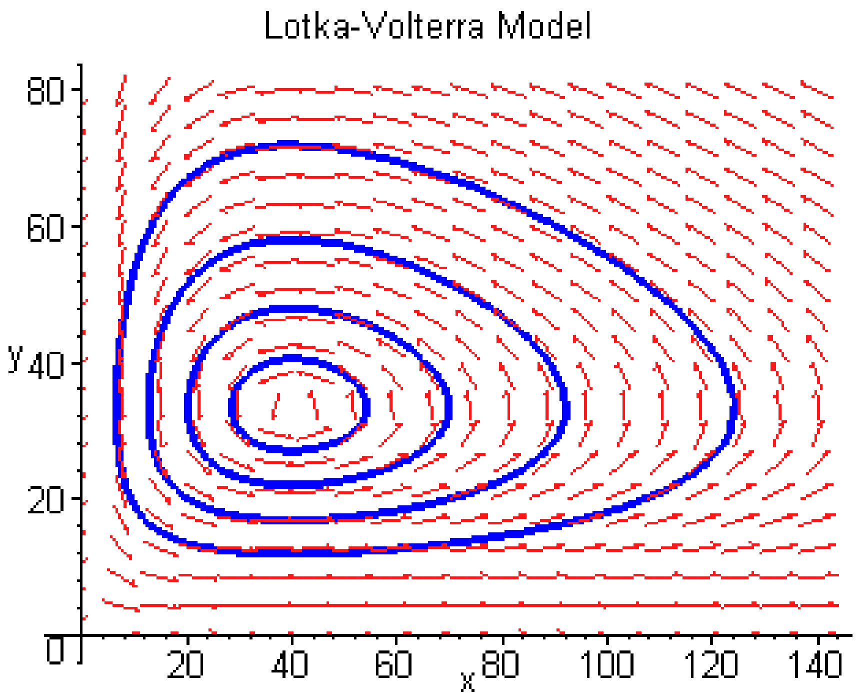


Figure 5.01


8.6 Appendix F: Acknowledgements

We would like to thank Tom Swiler for his assistance during the coding of this project, and Jim Mims for putting in extra time to assist us with our presentations. Also, we would like to thank the judges of the Supercomputing Challenge for giving us constructive criticism which helped us improve our project and give us clearer goals.

8.7 Appendix G: Works Cited

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