Sparking It: Electric Fields of the Tesla Coil

New Mexico Supercomputing Challenge
Final Report
April 2, 2008

Team 63
McCurdy School

Team Members:

Sarah Garduno
Benjamin Dozier (Los Alamos High School)
Brandon Ramirez
Jacob Romero
Francisco Vigil
Miquela Trujillo

Teacher:
Robert Dryja

Project Mentors:
Robert Robey
Charles Burch
Darrin Visarraga
Randy Bos

Sparking It: Electric Fields of the Tesla Coil

Executive summary

Developed Model for Design of Discharge Tip

We modeled various points on the spike with Mathematica. In this program we varied the number of points used and their position. These points were used to calculate the force of the electric field. Using this program we were able to calculate the most likely area which the discharge would take place.

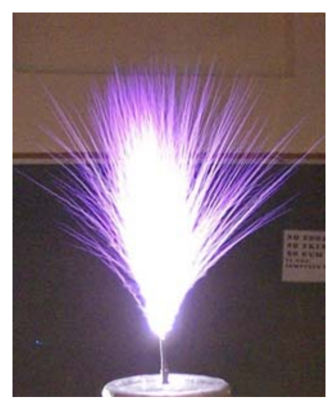
Developed Mathematical Model

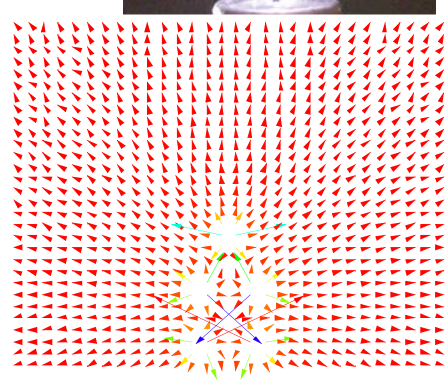
We used Maxwell's Equations to create our mathematical model. To calculate the force of the electrical field and the discharge we used Coulomb's Law which is derived from Gauss' Law. Coulomb's Law is applied to our project by using the formula to determine the magnitude and direction of the field and area of discharge. These calculations show that the electrical discharge will release in a conical form.

Matched Experimental Data

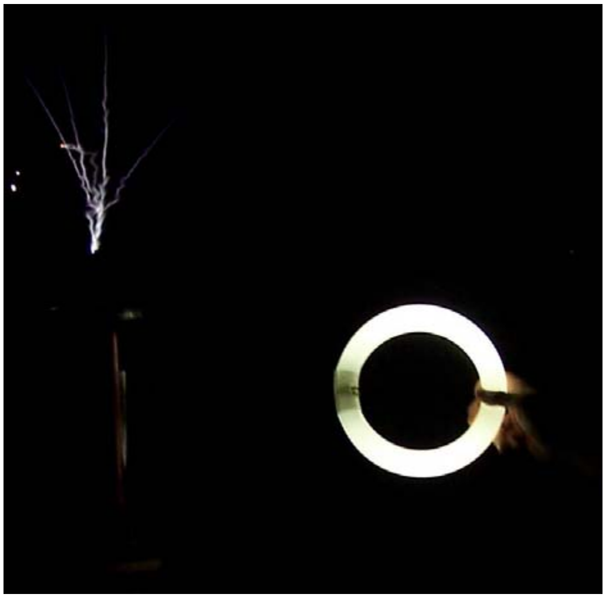
An operational Tesla Coil was used to physically verify our computational and mathematical models. We photographed the electrical discharge from the spike and compared it to our results from the computational and mathematical models. The photographs matched our models and showed that the discharge from the spike would occur in a conic shape.

1. Developed Model for the Design of Discharge Tip
2. Developed Math Model
3. Matched Experimental Data





Statement of the Problem



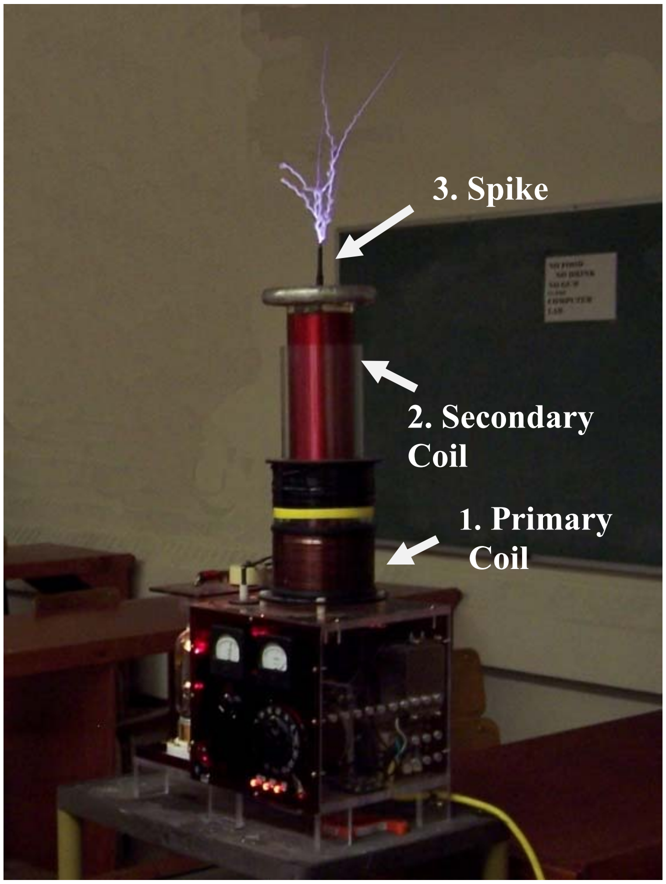
The problem addressed by our project was, can you create wireless electricity, and furthermore can you map the force of the electromagnetic fields that are created in the process of creating wireless electricity? First we had to find a device that would emit wireless electricity. It had to be able to create an electromagnetic field that would enable another electric device to work wirelessly or without a wired source. We also need to create a program that would accurately display the force or magnitude and direction of the electromagnetic field and have it proved by the engineered model. The process of creating such a model would prove to be harder than simplified above.

Solutions to the Problem

Our teacher, Robert Dryja, suggested a Tesla coil as a solution to our problem of generating wireless electricity. A Tesla coil is a high voltage, high frequency generator that is used in modern day experiments to observe alternating electricity. After extensively researching Tesla coils we realized that they would be an appropriate resolution to our problem and because of prior experiments and examinations of Tesla coils we knew that they could successfully produce what we wanted; a way to help show the force of the fields, suitable proof of wireless electricity, and an engineering challenge.

Mathematical Model

Applications of Maxwell's Equations



Maxwell's Equations encompass all of the mathematics involved in an operational Tesla Coil. Maxwell's Equations describe the correlation between the electric fields, magnetic fields, electric charge and electric current. These equations are used to calculate the dimensions of the electromagnetic fields created by a Tesla Coil. They also calculate the electric current as it travels through the device. Potential energy, which creates the discharge, is also calculated by these equations.

Maxwell's Equations as Applied to the Various Sections of the Tesla Coil

All eight of the Maxwell's Equations are required to calculate the dimensions of the primary and secondary coils. The Primary and Secondary Coils, consist of electric currents, electric fields and magnetic fields. All of these aspects are interconnected with all of the equations.

Model of the Electrical Discharge from the Spike

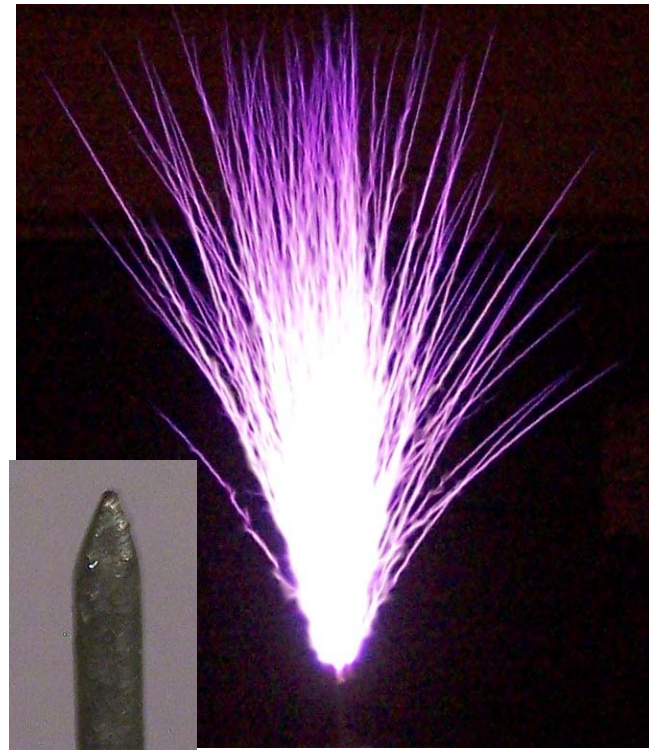
We are modeling the electric discharge from the spike on the top of the Tesla Coil. To calculate the potential energy from the electrical discharge only Gauss' Law is needed. Gauss' Law calculates the potential energy of the surface area of the spike which creates the electrical discharge. Gauss' Law is the only equation required because the discharge does not consist of an electric or magnetic field. Gauss' law is the partial derivative of the electric displacement $\frac{C}{m^2}$ equals the electric density which is measured in coulombs per meter squared $\frac{C}{m^2}$ or $\nabla \cdot D = \rho$.

Maxwell's Equations

The law of total currents	$j_{tot} = j + \frac{\partial D}{\partial t}$
Definition of the magnetic vector potential	$\mu H = \nabla x A$
Ampère's circuital law	$\nabla x H = j_{tot}$
The Lorentz force	$E = \mu v \, x \, H - \frac{\partial A}{\partial t} - \nabla \phi$
The electric elasticity equation	$E = \frac{1}{\varepsilon} D$
Ohm's law	$E = \frac{1}{\sigma} j$
Gauss' law	$\nabla \cdot \mathrm{D} = \rho$
Equation of continuity of charge	$\nabla \cdot \mathrm{j} = -\frac{\partial \rho}{\partial t}$

Electrical Discharge Model

Why is Coulomb's Law applied?



Above is a picture of the electrical discharge from the Tesla Coil barrowed from Blue Sky Learner. On the lower left side is a close up picture of the spike of the Tesla Coil.

Coulomb's Law is derives from Gauss' Law which calculates the magnitude of the force from the electrical discharge. Coulomb's Law is applied to our project by using the formula to determine the magnitude and direction of the discharge. Coulomb's Law is $F = \frac{kq_1q_2}{r^2}$ or Force equals a constant times point charge one times point charge two. The constant is approximately $9x10^9$ Newton times meter squared over coulombs squared. The point charge q_1 and q_2 are various points on the spike. r^2 is the distance between the two points squared. Coulomb's Law is applied to determine the vector's magnitude. These calculations show the discharge in a conical form.

Coulomb's Law

$$F = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$

Variables for Coulomb's Law

F= force
Coulomb's constant = k
k=$4\pi\varepsilon_0 \approx 9x10^9 N\cdot\frac{m^2}{C^2}$
q_1 = point charge
q_2 = point charge
r^2= distance between the two points.

Importance of the Shape

The shape of the spike is vital to the shape of the field as well as the magnitude and direction of the area of the discharge. It is a characteristic of electrical fields to discharge at the weakest point. For example, if a rod with a blunt end was used, the electrical energy would build up until discharged where the weakest point was. The same principle applies to a spherical tip. In a spherical tip, more energy would have to be built up than in a pointed tip because there are fewer imperfections or weak points in a sphere for the electrical energy to discharge from. A point or tip is geometrically the weakest point so electricity always discharges through the tip of the spike.

Limitations

In our calculations we are not including several variables which we cannot readily control. These variables may or may not have an effect on the electrical discharge. We did not take into account the effect of the magnetic field on the electrical discharge. The primary and secondary coils create electromagnetic fields which may have an effect on the discharge; however we were unable to model or calculate its impact. With our calculations we are assuming that the air in which we are testing the electrical discharge will not be ionized and the humidity will not affect the field or the effect will be equally distributed and all equally affected. Facilities were not available to replace air with Sulfur Hexafluoride, which would create a neutral environment. We can only calculate the area in which the discharge will occur by modeling the electric field. We are not able to calculate an exact path for the electrical discharge. Various interchangeable components can change the efficiency of the Tesla Coil and its fields. We are assuming that the material of the spike is completely uniform with no chemical or physical imperfections. Conductance of the material used is not factored in either.

Simplification	Justification
Magnetic fields do not influence the electrical discharge	The magnetic field does not affect any aspects of the discharge in which we are measuring.
Air does not affect the field	We cannot able to replace air with Sulfur Hexaflouride.
Cannot calculate an exact path for the electrical discharge	Can only simulate the area in which the electricity with discharge into.
The material of the spike is uniform	Cannot calculate the conductance of the various materials

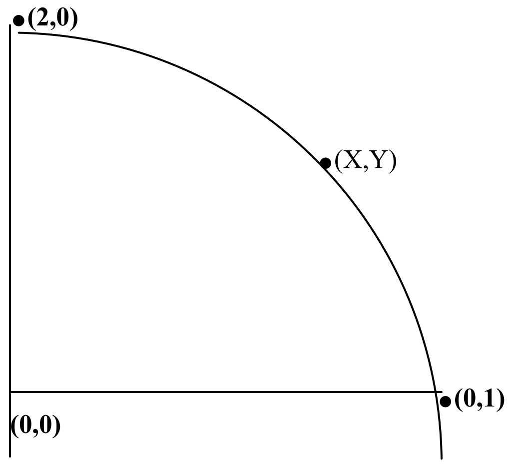
Computational Math for electrical fields: An Algebraic Model

Coulomb's law is the equation used to calculate the amount of force that one electron has on another. In our project we have decided to calculate the magnitude of the electric field as well as calculate the electric field as though the electrons are following an ellipse pattern. The amount of mathematical computing that would be required to calculate several points would be endless. We decided to start off with our mathematical model by using five points that will be on an ellipse form. The properties of an ellipse state that one side is a reflection of the other which means we can simplify our problem to three points. One point will be on the vertex, one on the origin and one that is equal distance from the first two points.

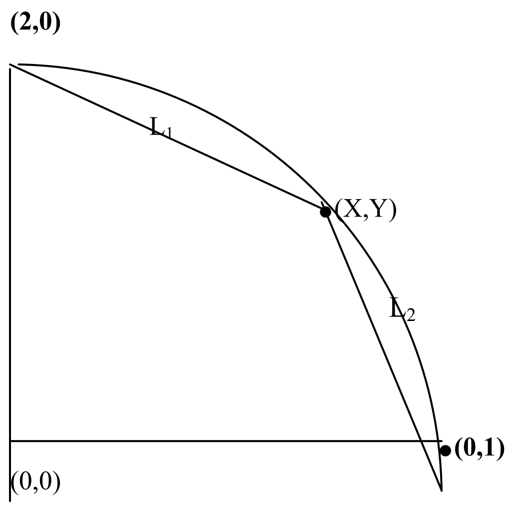
The beginning of our ellipse is at the end of the spike on the toroid. The electrical discharge that will come off the spike starts in an upward path that begins to curve as the spike increases in the upward direction. This forms an ellipse that will contribute to mapping out the direction of the electric field. One of the points that will be used in Coulomb's Law equation is located at the origin of the spike, the second point that will be used is the point where the electron is about to change directions of the path creating the ellipse. This point will be the vertex. In order to find the next point on the ellipse however, there are several other points that need to be located to plot out the electric field. There are two known electrons, but to begin the calculating we need to find the third point. We began this process by graphing out our points and ellipse. Our origin point is (0,1) and our vertex point is (2,0). We want to find a point that is equal distances from both the vertex and the origin point. One point is placed in the middle of the two points and will named (X,Y) being that we do not know the coordinates of this new point. Refer to **Step 1** for visual aid on the following page.

Once we placed the third point we drew a straight line from the electron on the vertex to the electron that is known as (X,Y). This segment is labeled as L_1. The same technique was used between the electron found on the origin as well as the electron labeled (X.Y). This second line segment is labeled as L_2. Refer to **Step 2** for visual aid on the following page. Once these two lines were drawn we had to draw a segment, that connects the middle electron with the other electrons. There is a horizontal line that leads to the vertex point, but if we connect the (X,Y) electron with this segment we have come up with segment X. This creates a smaller 90° triangle. The same technique is done the electron found on the origin and the mid point. The line that is now vertically connecting the mid point and the origin point creates another 90°. This vertical line will be labeled as Y. Refer to **Step 3** for visual aid on the following page. As we look at our graph we have three right angles and this allows us to use a series of calculations similar to the Pythagorean Theorem to solve for (X,Y).

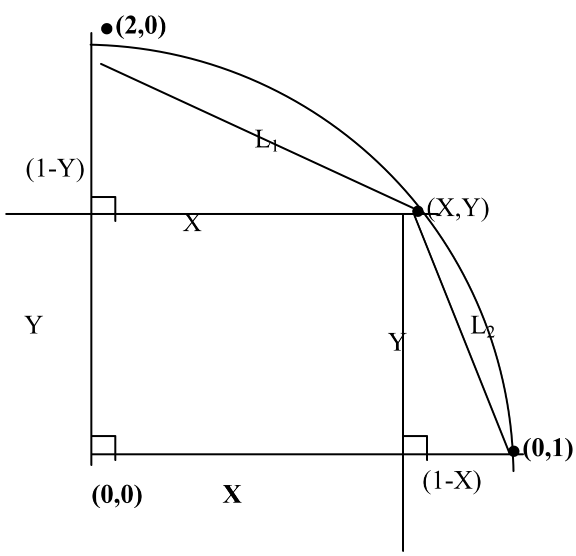
(Step1)



(Step 2)



(Step 3)



This first series of calculations will start with finding the length of the L_1. The opposite length of the 90° is 2-y because of the fact that we need to keep in mind the larger angle that we had in the beginning of our process which was 2. How we came up the formula of 2- y was because the opposite side of the smaller triangle was y but because it is a part of the larger triangle we needed to subtract y from 2. Now we have the opposite length as well as the adjacent length and we can now find the formula by using Pythagorean Theorem. This formula will come out to being $L_1= \sqrt{(x)^2 + (2-y)^2}$. This is the first formula that we will need to find (X,Y).

Our second series of calculations will be finding the length of L_2. The opposite length of the 90° is 1-x because of the fact that we need to keep in mind the larger angle that we had in the beginning of our process which was 1. How we came up the formula of 1- x was because the opposite side of the smaller triangle was x but because it is a part of the larger triangle we needed to subtract x from 1. Now we have the opposite length as well as the adjacent length and we can now find the formula by using Pythagorean Theorem. This formula will come out to being $L_2= \sqrt{(x)^2 + (1-x)^2}$. This is the second formula that we will need to find (X,Y).

Pythagorean Theorem
$c^2= a^2+b^2$
or as applied here
$L_1= \sqrt{(x)^2 + (2-y)^2}$

Now that we have our two formulas we are able to equal both equations to each other because the two segments, L_1 andL_2, are equal to each other. When the equations became equal to each other the process of finding the square root of both equations becomes cancelled out and no longer necessary to calculate. **The new equation is now $X^2+ (2-y)^2 = (1-x)^2+y^2$**. Now we have one equation and two unknowns. The easiest way to solve this problem is by solving for a system of equations by using the equation for solving an ellipse.

By using both of these formulas we can change the variables A and B in the ellipse formula to change the distance between the points on our ellipse. We only have to do the first steps of our mathematical problem (as seen above from step one to step three) because of how we set up the equation with two unknowns. **The ellipse formula is $X^2/A + Y^2/B = 1$.**

In order to check to see if our equations would be successful in finding our (X, Y) values we substituted numbers for the variables A, B. The numbers we chose were 1 and 4. When we began calculating the equations we came up with this problem. $\mathbf{\frac{X^2}{1} + \frac{Y^2}{4} = 1}$ We wanted to make the equation simpler so we multiplied **all sides by four. Our new equation is $4x^2 + y^2 = 4$**. As mentioned above we wanted to solve for (X,Y) by using the system of equations method which meant that we needed to get Y by itself in order to continue with our math problem. To get there we subtracted **$4x^2$ from both sides which left us with $y^2 = 4 - 4x^2$**. There is still one more step to finishing this portion of the problem which is getting the square root of $4 - 4x^2$. **The final formula for y is $y = \sqrt{4 - 4x^2}$**. By finding the y value using the ellipse formula we can go to our original equation and solve for X.

Previous Equation Applied
(on previous page)
$X^2 + (2-y)^2 = y^2 + (1-x)^2$

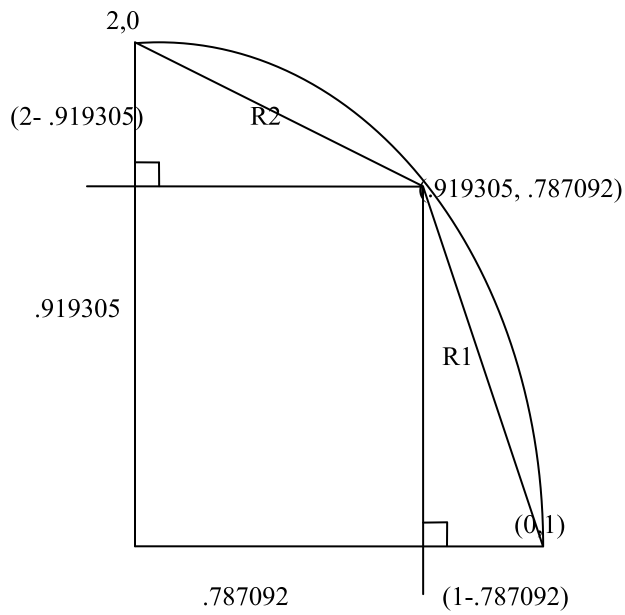
Before we substitute the y value from the ellipse formula however we must first simplify the one equation with two unknowns first. The equation we started off with is $X^2 + (2-y)^2 = y^2 + (1-x)^2$. When we multiply out the middle functions i.e ((2-y)(2-y)) and our new solution is $4-4y+y^2$. We then go and do the same for the X function i.e ((1-X)(1-X)). The new solution for the X function is $1-2x+x^2$. Once we have simplified these two functions we are able to continue to the main math problem.

The new equation after all the simplifications we have made is as follows:

$-x^2+4-4y+y^2=y^2+1-2x+x^2$. From this equation we continued, by hand, calculating the value of x. As soon as we got close to the solution we were faced with a polynomial that contained 4 orders, 3rd orders and 2nd orders. Being that the polynomial is of a fourth order we realize that our solution will contain four answers, two unreal solutions and two real solutions. Also the math that we were faced with in this one problem we realize that by getting the square roots of those numbers that we would have two solutions, a negative and positive but because we are solving for length and there can be no negative lengths. We base our calculations on the positive solutions. We could continue to solve by hand but the work would be so tedious that we decided to make use of Mathematica. On Mathematica, we were able to plug in this equation $-x^2+4-4y+y^2=y^2+1-2x+x^2$ and solve for x. The answer in which we got from Mathematica, **for the X value was .919305**. Now that we have gotten x we can go back to the ellipse formula and plug in the X value and solve for Y. Again for this portion of the math problem we referenced Mathematica and **were given the solution of .0787092 for Y**. Now we have our (X, Y) values of our third point on our ellipse. Before we can begin working with Coulomb's Law, to calculate the amount of force that each electron is having on each point, we must first find the distance between each electron.

The r variable in Coulomb's Law is simply the distance between each electron. Once again we were able to use the Pythagorean Theorem to find r. See **Figure (1-1)** for visual aid on the following page.

Figure (1-1)



Once again we have the original two triangles but in this figure the values of X, and Y are found. By having these solutions we can use the Pythagorean Theorem to solve for R^1 and R^2. The formula for finding R^2 is $R=\sqrt{(.919305)^2+(2-.919305)^2}$. The equation for finding R^1 is $R=\sqrt{(..787092)^2+(1-.787092)^2}$. We now have the R values and start working with Coulomb's Law. With Coulomb's Law we simplified the equation to where it was:

$$\mathbf{F = \frac{q_1^{\,2}}{R^2}}$$

We were able to make this simplification because the distances between the two were the same. We also eliminated the variable of the constant because of the fact that the Constant did not affect our solutions at all.

Assumptions	Justifications
L1=L2	The right angle as well as the fact that the mid point is equal distance from both the vertex and origin.
$F = \frac{q_1^2}{R^2}$	The equal distance between the tow lengths are the same which means r can b to the second power. Also we can simplify q1 because of the fact the electrons have the same charge and will not change.

Once we find the amount of force that each electron has one another we could then go in and a do a simple vector analysis. In the vector analysis we can calculate these two ways, algebraically or graphically. Refer to **Figure (2-1)** and **(2-2)** on the following pages.

Figure (2-1)

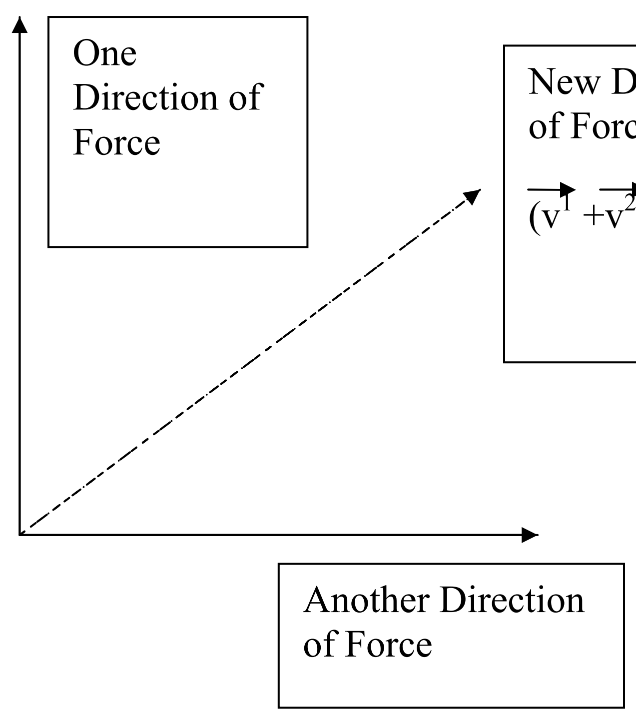


Figure (2-2)

$\vec{V}^1 = V_1\,Xi + V_1Yi$

$\vec{V}^2 = V_2\,Xj + V_2Yj$

Sum = $(\vec{v}^1 + \vec{v}^2) = (V_1\,x + V_1y)I + (V^2x + V^2y)J$

Magnitude = $|(\vec{v}^1 + \vec{v}^2)| = \sqrt{}\,(V_1\,x + V_2x)^2 + (V_1y + V_2y)^2$

In Microsoft Excel we were able to work with the ellipse formula to calculate some graphs that shows the path that our possible equations may form. We basically changed the A values around and had the distance between the points equal of .5. An example of our Excel Spread Sheet is as follows **Figure (3-1)**. To see the graphs that we calculated please see **Figure (3-2)** and **Figure (3-3)** on the following pages.

Figure (3-1)

Formula for an ellipse with a vertical major axis:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Example:

Let the selected x value be limited to x < -1 and x < 1
Let the major axis have a = **1**, **2**, or **3.**
Have the minor axis have b = 1

Let a > b

The calculating formula then is

$$\frac{y^2}{a^2} = 1 - \frac{x^2}{b^2} \quad \text{or} \quad y^2 = a^2\left(1 - \frac{x^2}{b^2}\right)$$

$$\text{or} \quad y = \sqrt{a^2\left(1 - \frac{x^2}{b^2}\right)}$$

Pseudo Mathematica Program

Ellipse calculations for Three Values of A Focus. r^2 not equal. Use Sarah/Jacob math to obtain equal r^2 values

a	b	x	y	a	b	x	y	a	b	x	y	x	y
3.00	1.0	-1.00	0.000	**2.00**	1.0	-1.00	0.000	**1.00**	1.0	-1.00	0.000	1.00	0.40
3.00	1.0	-0.95	0.937	2.00	1.0	-0.95	0.624	1.00	1.0	-0.95	0.312	1.05	0.55
3.00	1.0	-0.90	1.308	2.00	1.0	-0.90	0.872	1.00	1.0	-0.90	0.436	1.10	0.70
3.00	1.0	-0.85	1.580	2.00	1.0	-0.85	1.054	1.00	1.0	-0.85	0.527	1.15	0.85
3.00	1.0	-0.80	1.800	2.00	1.0	-0.80	1.200	1.00	1.0	-0.80	0.600	1.20	1.00
3.00	1.0	-0.75	1.984	2.00	1.0	-0.75	1.323	1.00	1.0	-0.75	0.661	1.25	1.15
3.00	1.0	-0.70	2.142	2.00	1.0	-0.70	1.428	1.00	1.0	-0.70	0.714	1.30	1.30
3.00	1.0	-0.65	2.280	2.00	1.0	-0.65	1.520	1.00	1.0	-0.65	0.760	1.35	1.45
3.00	1.0	-0.60	2.400	2.00	1.0	-0.60	1.600	1.00	1.0	-0.60	0.800	1.40	1.60
3.00	1.0	-0.55	2.505	2.00	1.0	-0.55	1.670	1.00	1.0	-0.55	0.835	1.45	1.75
3.00	1.0	-0.50	2.598	2.00	1.0	-0.50	1.732	1.00	1.0	-0.50	0.866	1.50	1.90
3.00	1.0	-0.45	2.679	2.00	1.0	-0.45	1.786	1.00	1.0	-0.45	0.893	1.55	2.05
3.00	1.0	-0.40	2.750	2.00	1.0	-0.40	1.833	1.00	1.0	-0.40	0.917	1.60	2.20
3.00	1.0	-0.35	2.810	2.00	1.0	-0.35	1.873	1.00	1.0	-0.35	0.937	1.65	2.35
3.00	1.0	-0.30	2.862	2.00	1.0	-0.30	1.908	1.00	1.0	-0.30	0.954	1.70	2.50
3.00	1.0	-0.25	2.905	2.00	1.0	-0.25	1.936	1.00	1.0	-0.25	0.968	1.75	2.65
3.00	1.0	-0.20	2.939	2.00	1.0	-0.20	1.960	1.00	1.0	-0.20	0.980	1.80	2.80
3.00	1.0	-0.15	2.966	2.00	1.0	-0.15	1.977	1.00	1.0	-0.15	0.989	1.85	2.95
3.00	1.0	-0.10	2.985	2.00	1.0	-0.10	1.990	1.00	1.0	-0.10	0.995	1.90	3.10
3.00	1.0	-0.05	2.996	2.00	1.0	-0.05	1.997	1.00	1.0	-0.05	0.999	1.95	3.25
3.00	1.0	0.00	3.000	2.00	1.0	0.00	2.000	1.00	1.0	0.00	1.000	2.00	3.40
3.00	1.0	0.05	2.996	2.00	1.0	0.05	1.997	1.00	1.0	0.05	0.999	2.05	3.55
3.00	1.0	0.10	2.985	2.00	1.0	0.10	1.990	1.00	1.0	0.10	0.995	2.10	3.70
3.00	1.0	0.15	2.966	2.00	1.0	0.15	1.977	1.00	1.0	0.15	0.989	2.15	3.85
3.00	1.0	0.20	2.939	2.00	1.0	0.20	1.960	1.00	1.0	0.20	0.980	2.20	4.00
3.00	1.0	0.25	2.905	2.00	1.0	0.25	1.936	1.00	1.0	0.25	0.968	2.25	4.15
3.00	1.0	0.30	2.862	2.00	1.0	0.30	1.908	1.00	1.0	0.30	0.954	2.30	4.30
3.00	1.0	0.35	2.810	2.00	1.0	0.35	1.873	1.00	1.0	0.35	0.937	2.35	4.45
3.00	1.0	0.40	2.750	2.00	1.0	0.40	1.833	1.00	1.0	0.40	0.917	2.40	4.60
3.00	1.0	0.45	2.679	2.00	1.0	0.45	1.786	1.00	1.0	0.45	0.893	2.45	4.75
3.00	1.0	0.50	2.598	2.00	1.0	0.50	1.732	1.00	1.0	0.50	0.866	2.50	4.90
3.00	1.0	0.55	2.505	2.00	1.0	0.55	1.670	1.00	1.0	0.55	0.835	2.55	5.05
3.00	1.0	0.60	2.400	2.00	1.0	0.60	1.600	1.00	1.0	0.60	0.800	2.60	5.20
3.00	1.0	0.65	2.280	2.00	1.0	0.65	1.520	1.00	1.0	0.65	0.760	2.65	5.35
3.00	1.0	0.70	2.142	2.00	1.0	0.70	1.428	1.00	1.0	0.70	0.714	2.70	5.50
3.00	1.0	0.75	1.984	2.00	1.0	0.75	1.323	1.00	1.0	0.75	0.661	2.75	5.65
3.00	1.0	0.80	1.800	2.00	1.0	0.80	1.200	1.00	1.0	0.80	0.600	2.80	5.80
3.00	1.0	0.85	1.580	2.00	1.0	0.85	1.054	1.00	1.0	0.85	0.527	2.85	5.95
3.00	1.0	0.90	1.308	2.00	1.0	0.90	0.872	1.00	1.0	0.90	0.436	2.90	6.10
3.00	1.0	0.95	0.937	2.00	1.0	0.95	0.624	1.00	1.0	0.95	0.312	2.95	6.25
3.00	1.0	1.00	0.134	2.00	1.0	1.00	0.089	2.00	1.0	1.00	0.089	1.00	6.40

Figure (3-2)

This graph represents the ellipse calculations from preceding Figure 3.1. The ellipse formula is modified so that equal r^2 lengths are calculated rather than the arbitrary values used initially.

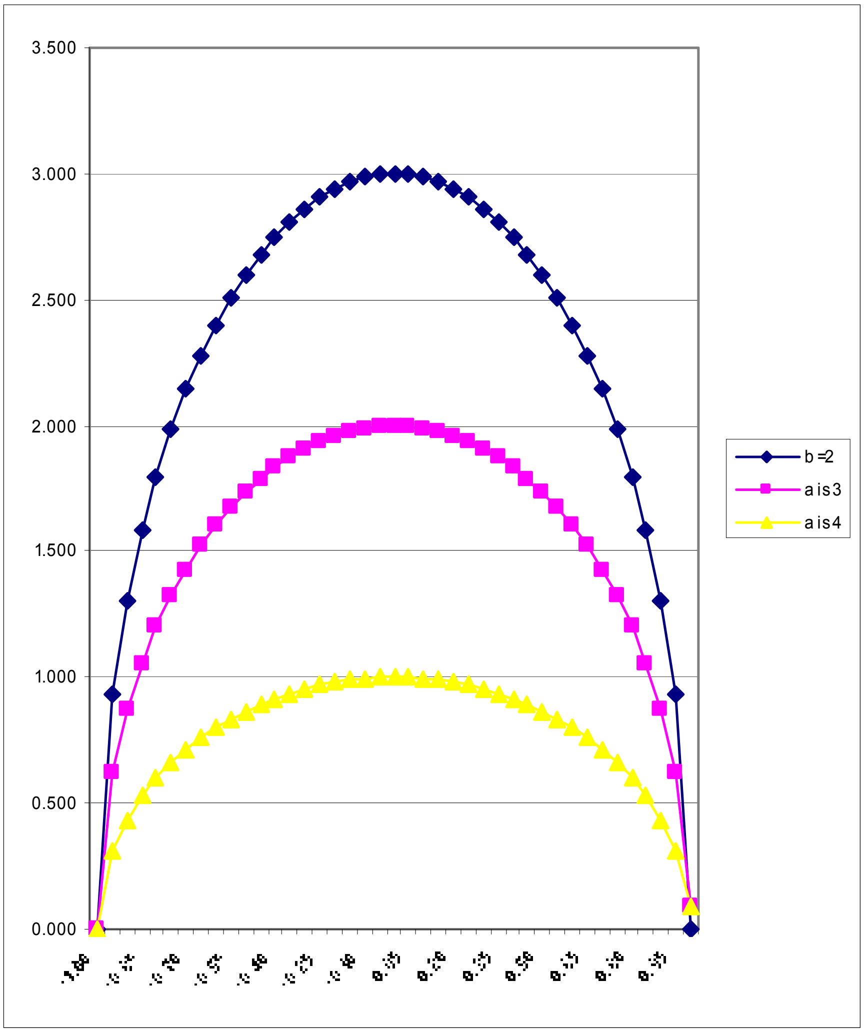
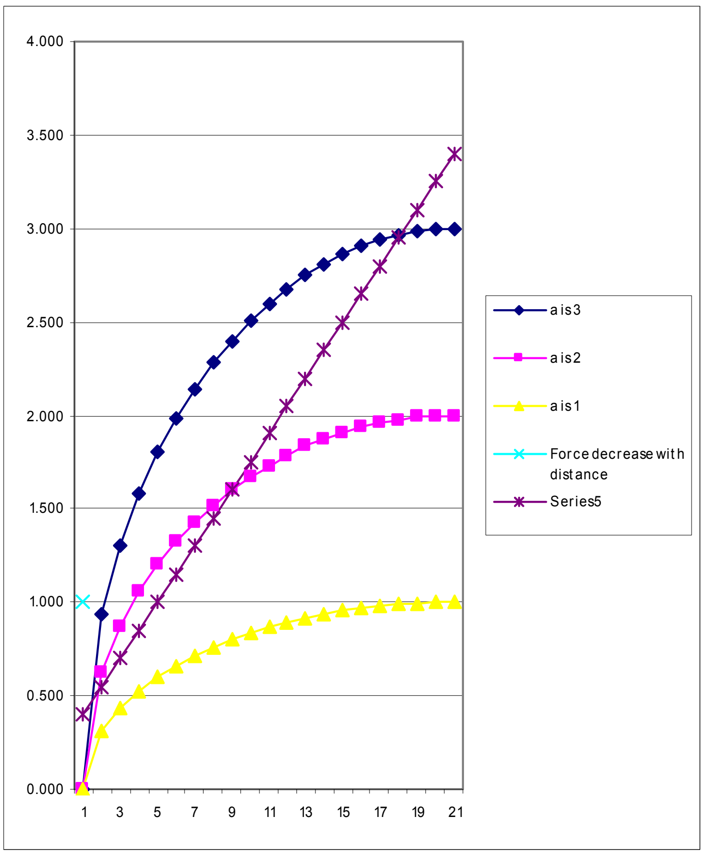


Figure (3-2)

This graph represents both the ellipse calculations as well as the magnitude and force calculations. The same as for Figure 3.1, the ellipse formula is modified so that equal r^2 lengths are calculated rather than the arbitrary values used initially.



Programming and Code with Mathematica

Language

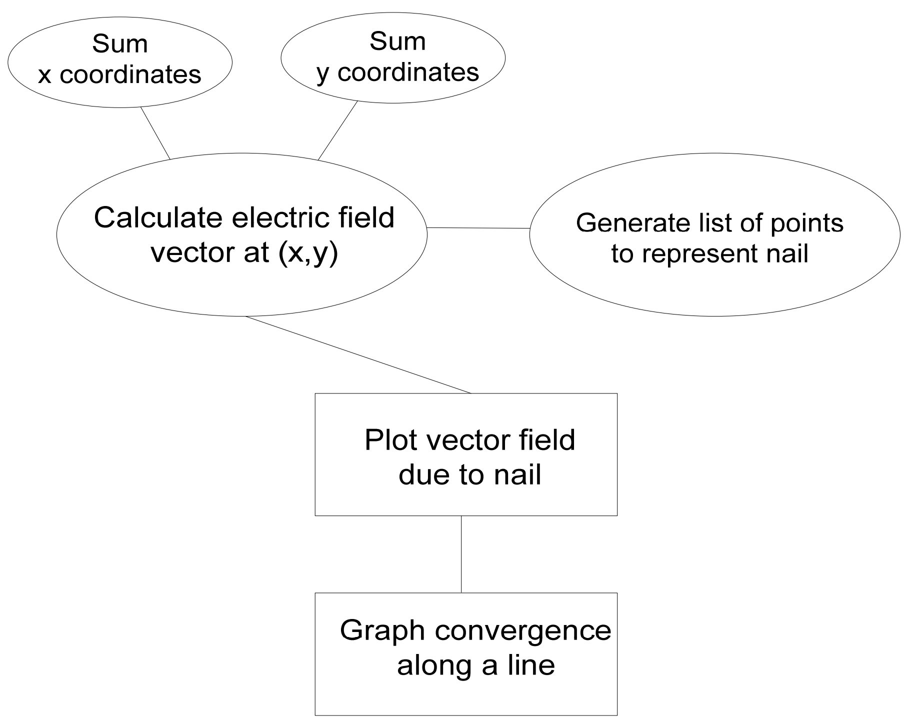
Our model was written and implemented using the Wolfram Mathematica environment. We chose this package because of its powerful graphics capabilities and ease of use. Additionally, team members had some familiarity with Mathematica prior to the start of the project. All of the code is original, though certain parts may utilize optimizations that are built into the Mathematica environment. The script is approximately 55 lines long.

Preliminaries

First we write Coulomb's law in vector form which will allows us to easily calculate the electric field vector at a point:

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q\vec{r}}{\|\vec{r}\|^3}}$$

Flowchart



Code Snippet

```
fieldpoints[x_,y_,pointlist_]:=(
(* Given a point (x,y) and a list of point charges, this function
calculates the electric field at (x,y) *)

     xcoord=Sum[(x-Part[Part[pointlist,i],1])/((x-
Part[Part[pointlist,i],1])^2+(y-
Part[Part[pointlist,i],2])^2)^(3/2),{i,Length[pointlist]}];
(* xcoord stores the value of the x-component of the electric
field vector *)

     ycoord=Sum[(y-Part[Part[pointlist,i],2])/((x-
Part[Part[pointlist,i],1])^2+(y-
Part[Part[pointlist,i],2])^2)^(3/2),{i,Length[pointlist]}];
     (* ycoord stores the value of the y-component of the
electric field vector *)

     {xcoord,ycoord}
     );
```

Graphical Output of the Program

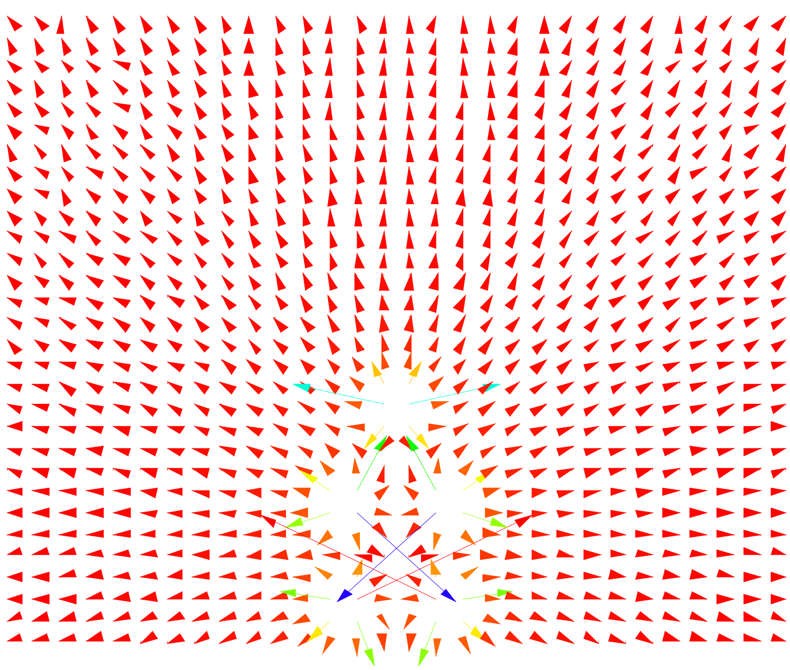


Figure 4-1: Nail with 5 points

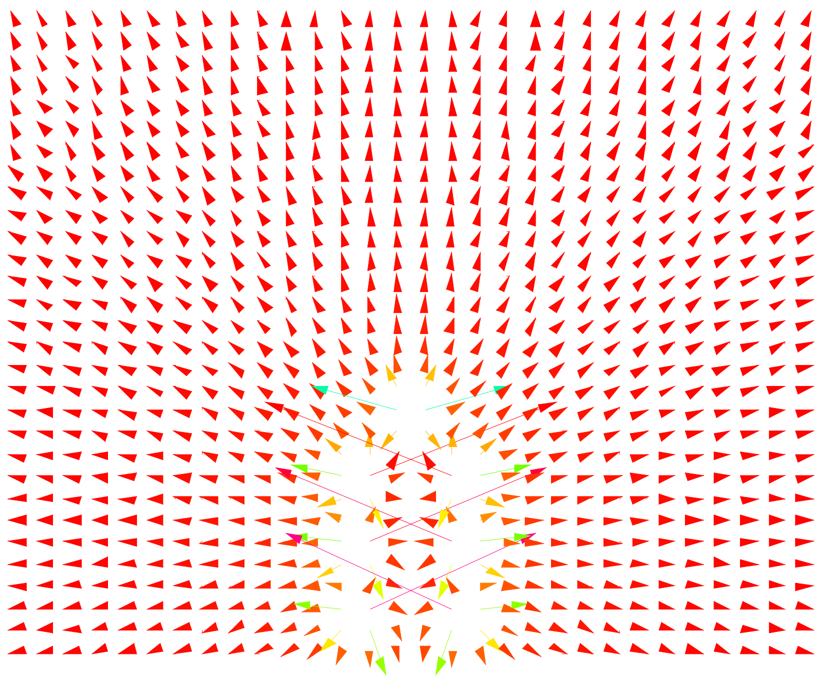
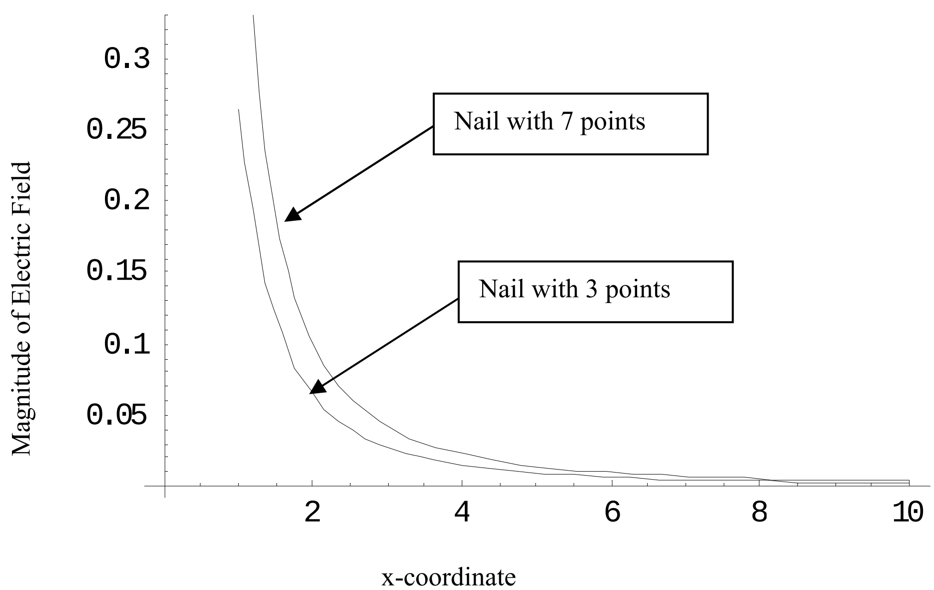


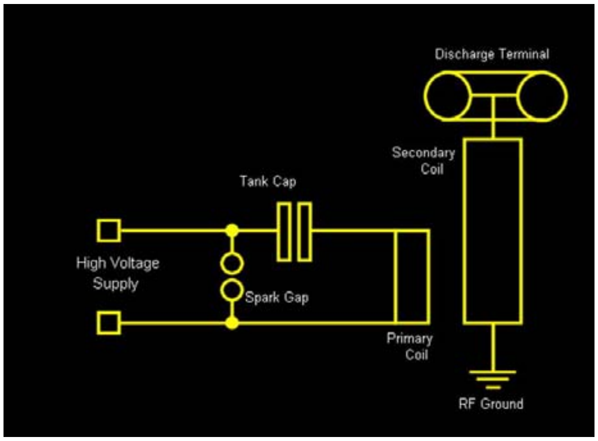
Figure 4-2: Nail with 7 points

Convergence

The second part of the program plots the magnitude of the field along the line *y=x-1*, which runs through the center of the nail.



Description of the methods used: Engineering: Why we need a physical model



A physical model allowed us to verify our mathematical and computational models. During the process of constructing a Tesla Coil we gained a better understanding of how they operate. We were able to visibly compare what we modeled with the electrical discharge which we photographed. Our Tesla Coil did not operate properly so we conducted our testing on a Tesla Coil which was lent to us by Big Sky Learning.

Variac-The variac allows us to slowly supply power to the transformer and the Tesla Coil. This minimizes the risk of shorting out components which are not properly insulated.

Transformer- The transformer is a 12,000 volt Neon Sign Transformer. The transformer converts the electric current from AC to DC and steps it up to 12,000 volts.

Secondary Coil-The secondary coil acts as a transformer and creates an electromagnetic field. Electrical energy which is transferred from the primary is stepped up from the original 12,000 volts through induction. It is not physically connected to the primary coil so all electrical energy is transferred between their fields. To have an efficient transfer of energy, the primary must have the same resonant frequency as the primary coil.

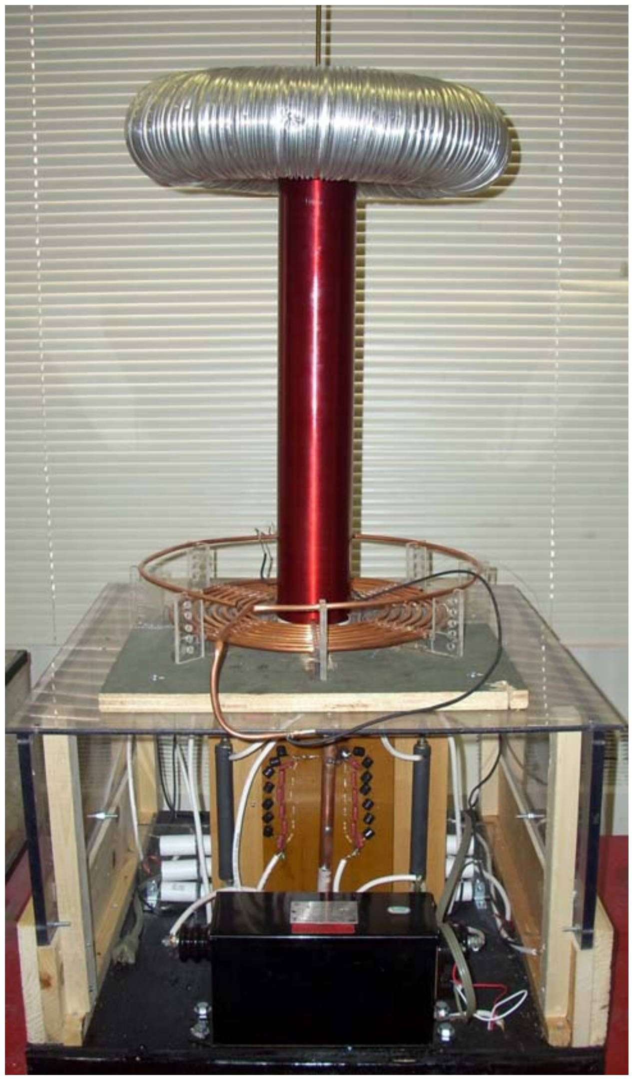
Circuit Board-The circuit board is a safety device. It contains a safety gap which is connected to a ground. An array of diodes and capacitors protect the other components in the Tesla Coil from being shorted out. Bleed resistors slowly discharge the capacitors to the ground.

Capacitor Bank (MMC)-The capacitor bank consists of 23, .15μF capacitors connected in series. The current travels from the transformer, through the circuit board, and into the capacitor bank. Once the capacitors full the energy is discharged through the spark gap and into the primary coil.

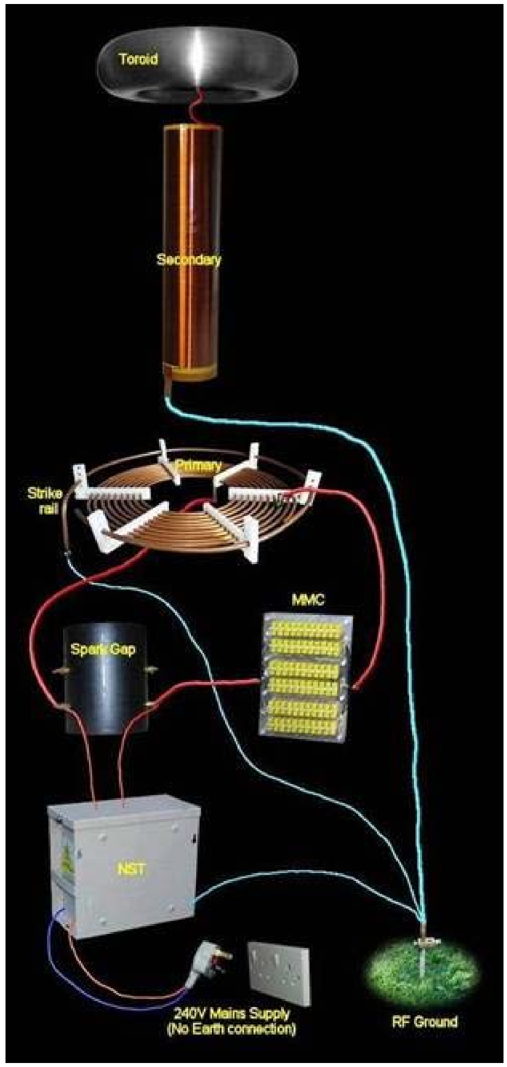
Spark Gap-The spark gap allows the energy in the capacitor banks to be built up until it can spark across the 1.4 inch gap. The length of the gap and the capacitance of the capacitors create pulse of electricity which cycles at 60 Hz through the primary coil.

Primary Coil-The primary coil emits an oscillating electromagnetic field which transfers electricity to the secondary coil.

Toroid-The toroid is attached to the spike where the electricity discharges from. It emits an electromagnetic field which channels the electrical discharge away from the secondary coil.



The Tesla Coil that we built



A Tesla Coil Diagram. Showing the electrical connections

Results & Conclusions

Finally, we developed an algebraic model to try to anticipate the path of the spark coming off the top of the spike. We also developed a computer model in Mathematica that also maps the projected area of the discharge, along with the direction and magnitude to develop the force of the electrical field. Both of the models were validated and supported by each other producing the desired results. They were further authenticated by the physical model of a Tesla coil through the time-lapse photographs which displayed the electrical discharge. We clearly succeeded in solving our problem by effectively constructing a Tesla coil, and mapping the electromagnetic fields given off by the Tesla coil.

Teamwork

Our team consists of six students; five from McCurdy High School and one from Los Alamos High School. Teamwork was a challenge for everyone on our team; however we all worked together and had certain responsibilities. We all worked together to involve everyone in our team in our different sections and made sure all of us were at the same level of understanding. We all worked together on the construction of the Tesla coil. However, we did distribute the work of writing the reports evenly. Here is some recollection of some of what the team members did. Sarah worked a lot on the computational model and the dimensions of a Tesla Coil. Jacob worked on the electrical engineering of the Tesla Coil. He also worked with Sarah on the computational model. Benjamin was our main programmer. He primarily worked on the program. Brandon helped Ben with the programming and was our editor for our reports. Francisco was one of the engineers for the construction of the Tesla Coil along with the mathematical model. Miquela was also an engineer working on the construction of the Tesla Coil. She also worked with Francisco on the mathematical model. Each team member had certain sections that we were in charge of. We all worked together in the engineering, construction, writing the report or helping in comprehension.

Future Plans

In the future we are planning on forming an additional code that will display the equations that we have come up with for calculating the electric field if the electrons followed an ellipse path. This Program will help because if we take the program that we have now illustrates force of the electrons in a vertical path we will be able to clearly display the electric field more accurately. We also are planning on doing more calculations with different variables for our equations. We feel as though the more points we will establish a form of convergence that will show how accurate our solutions are. We also would like to finish applying Coulomb's Law as well as beginning our vector analysis to see how the different distances would have an affect on our graph.

As for the computer model in future work we may focus on examining the magnitude of the electric field along elliptical paths going out from the center of the nail.

Most significant original achievement on the project

The most significant original achievement for our group would be that we were able to verify our computational and mathematical model with an operational Tesla Coil. This allowed us to understand the physical properties of electric fields and gain a deeper understanding of what the equations we used are capable of. Constructing the physical model also taught us the engineering work required for experimentation like this.

References

"Coulomb's Law- Law of Force". Science Joy Wagon.1999. <http://regentsprep.org/Regents/physics/phys03/acoulomb/default.htm>.

Inc., Wolfram Research. Mathematica for Students.2008 (Software).

Maxwell, James Clerk. "On Physical Lines of Force." 1861. Retrieved March 20, 2008. <http://en.wikipedia.org/wiki/Maxwell's_Equations>.

Sternway, Raymond.A , and Jerry S. Faughn. Physics. New York: Holt, Rinehart and Winston, 1999.

Visarraga, Darrin. Mathematician. Interview. March 28, 2008.

Appendix A: Mathematica Code

```
<<Graphics`PlotField`
(* loads the graphics package for plotting vector fields *);
Off[General::spell1];

fieldpoints[x_,y_,pointlist_]:=(
     (* Given a point (x,y) and a list of point charges, this function calculates the electric
field at (x,y) *)

     xcoord=Sum[(x-Part[Part[pointlist,i],1])/((x-Part[Part[pointlist,i],1])^2+(y-
Part[Part[pointlist,i],2])^2)^(3/2),{i,Length[pointlist]}];
     (* xcoord stores the value of the x-component of the electric field vector *)

     ycoord=Sum[(y-Part[Part[pointlist,i],2])/((x-Part[Part[pointlist,i],1])^2+(y-
Part[Part[pointlist,i],2])^2)^(3/2),{i,Length[pointlist]}];
     (* ycoord stores the value of the y-component of the electric field vector *)

     {xcoord,ycoord}
     );

GenerateNailList[k_]:=Join[Table[{-0.25,-1+i/k},{i,0,k-1}],Table[{0.25,-1+i/k},{i,0,k-
1}],{{0,0}}];
(* Generates a list of points that approximate the shape of the nail;
  k=number of points on each side; there are 2k+1 total points *)

PlotVectorField[fieldpoints[x,y,GenerateNailList[2]],{x,-2,2},{y,-1.2,2},ColorFunction→Hue,
ScaleFactor→1, PlotPoints→30];
(* nail = 5 points *)

PlotVectorField[fieldpoints[x,y,GenerateNailList[3]],{x,-2,2},{y,-1.2,2},ColorFunction→Hue,
ScaleFactor→1, PlotPoints→30];
(* nail = 7 points *)

PlotVectorField[fieldpoints[x,y,GenerateNailList[4]],{x,-2,2},{y,-1.2,2},ColorFunction→Hue,
ScaleFactor→1, PlotPoints→30];
(* nail = 9 points *)

P1=Plot[1/7 Norm[fieldpoints[x,x-1,GenerateNailList[1]]],{x,1,10}];
(* plots the magnitude of the electric field along the line y=x-1 *)

P2=Plot[1/11 Norm[fieldpoints[x,x-1,GenerateNailList[3]]],{x,1,10}];
(* plots the magnitude of the electric field along the line y=x-1 *)

Show[P1,P2];
```

Acknowledgments

Robert Robey for conducting meetings to explore the math and prepare this report

Charles Burch for helping to locating the parts for constructing the Tesla Coil

Darrin Visarraga for explaining and review the computational math

Randy Bos for reviewing the pure math

Big Sky Learning for loaning an operational Tesla coil while we were still constructing our own model