Housing Problem

New Mexico

Supercomputing Challenge

Final Report

April 10, 2024

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Over the past couple of decades, the prices of houses have gone up drastically. This makes it hard for people working on low wages to get a house. We want to be able to find optimal prices for both the seller and buyer.

We want accurate data pertaining to the current time. For housing prices, they are very different now than they were 20 years ago. Housing location is also important, with places like New York and San Francisco having high housing prices compared to Little Rock and Jackson. Choosing to generalize for the entire country, let alone the world, would not be beneficial as the data would only apply to the cities closest to the nation's average in price, space, and government funding [1], [2], [3]. Albuquerque is local and a good place to start, as even different places in New Mexico are going to be drastically different. In the future, we could find a way to more easily calculate prices for other cities as well as account for additional variables.

There are several ways current governments are trying to fix the housing problem, by using money generated from taxes the government can give out grants and loans for people to pay for housing, or build public housing, though this is hard because there are zoning laws and little space that isn't already being used [1]. Even if people receive money, this does not entirely solve the problem because there is often a shortage of houses anyway, especially in more populated cities. The United States Department of Housing and Urban Development (HUD) is the executive department in the federal government that is dedicated to housing. While it does do some to make public housing and shelter for people, and give out money, it has not nearly accounted for the rising house prices that have at least doubled [4]. This housing and money will also not reach everyone.

In order to find the best prices for houses we used the simplex method. The first thing I did was practice doing row echelon form and the simplex method on paper with some practice problems before starting on programming [5]. Then I wrote a program that would do “reduced row echelon form” to solve a system of inequalities and check if the answer it gave was correct [6]. That would be the basis for the simplex method that I wrote later. A python program was written that can do the simplex method for two variables. It later had the capability to do more variables. We took this code and converted it to C for future use on a supercomputer to process more data, faster[7]. The simplex method is what helped us find the cheapest house that can be built with the given constraints (space, resources, funding) that would also give enough money to the seller in order for them to buy a new one if they want.

1. To solve a set of equations using the simplex method, the first step is to set up the problem. This involves creating a maximizing/minimizing equation and constraints. Any real number can be used when creating one, but for the actual problem we used real world data from Albuquerque to make it.

$$\mathbf{Z = x_1 - 2x_2}$$

$$\mathbf{5x_1 + 2x_2 \leq 20}$$

$$\mathbf{5x_1 + 8x_2 \leq 50}$$

$$\mathbf{x_1, x_2 \geq 0}$$

2. The next step is getting the inequalities ready to be turned into a matrix. This will be done by adding slack variables to each inequality except for the maximizing/minimizing one to turn it into an equation and moving all the variables to one side of the equation.

$$Z - x_1 + 2x_2 = 0$$

$$5x_1 + 2x_2 + y_1 = 20$$

$$5x_1 + 8x_2 + y_2 = 50$$

$$x_1, x_2 \geq 0$$

3. These will be converted into a matrix (except for the non-negativity constraints).

x_1	x_2	y_1	y_2	Z	C
5	2	1	0	0	20
5	8	0	1	0	50
-1	2	0	0	1	0

Table #1. Equations put into a matrix

4. We then find the smallest negative number in the bottom row. Using the numbers from the row where the negative number lies, we can find the smallest quotient in the farthest row. We then save the number that lies in both the row and column we selected for later.

x_1	x_2	y_1	y_2	Z	C
5	2	1	0	0	20
5	8	0	1	0	50
-1	2	0	0	1	0

Table #2. Finding the smallest negative and quotient

5. Now we want to make the number we just found one so we divide everything in that row by that number.

x_1	x_2	y_1	y_2	Z	C
1	2/5	1/5	0	0	4
5	8	0	1	0	50
-1	2	0	0	1	0

Table #3. Dividing everything in the top row by five

6. Next, we want to make every other number in the same column zero. We do this by subtracting/adding the first row from each other row until the numbers in the same row are zero.

x_1	x_2	y_1	y_2	Z	C
1	2/5	1/5	0	0	4
0	6	-1	1	0	30
0	12/5	1/5	0	1	4

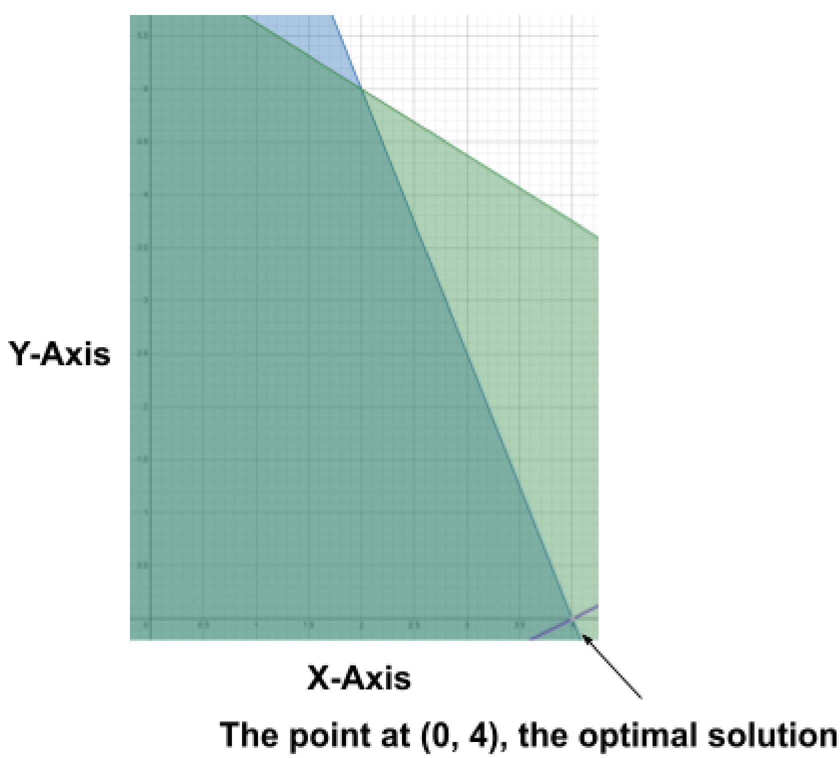
Table #4. Making every other entry in the first column zero

7. If there are still negative numbers on the bottom row, then go back to step four and go through the steps again until there are no negative numbers.
8. To get the results from the table, we want to find all the variables that have one one and the rest zero in their column. We can set the y_1 and x_2 to zero because they don't fit the requirements. X_1 is equal to four

We can now use real world numbers instead of random numbers in order to get meaningful results when choosing from a by using the simplex method.

Simplex Method in Graphing Form

This is done by taking our original constraints and graphing them [9], [10]. Anything within the shaded region (which would be below the line, but above the x-axis because of our nonnegativity constraints, also known as the feasible region). The optimal solution would be where the maximizing function intersects with the feasible region that creates the highest possible value. It most likely be on the edge of the feasible region (but could be anywhere in the feasible region if any point gives the optimal solution or anywhere on a specific edge if our maximizing/minimizing equation parallel to that edge). These are points you could try first.



[10]

Using graphing to find optimal solution

Four is also what we got for Z when solving the systems algebraically.

Equations used for graphing:

$$Z = x_1 - 2x_2$$

$$5x_1 + 2x_2 \leq 20$$

$$5x_1 + 8x_2 \leq 50$$

$$x_1, x_2 \geq 0$$

Our Results

Using real world numbers we can have quotations like this (Scaled down to smaller numbers and rounded):

$$Z = x_1 + 2x_2 + 3x_3 + 4x_4$$

$$205{,}500 \geq 1.2x_1 + 2.3x_2 + 2.5x_3 + 2.6x_4$$

$$200 \geq 1.05x_1 + 2.10x_2 + 3.15x_3 + 4.20x_4$$

$$205365 \geq 2.3x_2 + 2.6x_4$$

$$135 \geq 1.2x_1 + 2.5x_3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

x_1, x_2, x_3, x_4 are variables representing the different houses depending on how many people they house (one - four people). The first equation is our maximizing equation. We want to maximize the amount of people we can have. The first inequality is a constraint for the amount of space for housing [11]. The second inequality represents the amount of money that the government has [12], [13]. The Third and fourth constraints represent zoning, where the city has limited space to build houses of different sizes. I was not able to find the data specific zoning laws in

Albuquerque with numbers so these are made up in order to give good results (Having different numbers would result in just one type of house being built).

Putting these into our program and running it will give us these results:

x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	Z	C
0.2860	1.0000	0.0000	0.0000	1.0000	-0.619	0.0000	-0.220	0.0000	20535
-0.110	0.5000	0.0000	1.0000	0.0000	0.2381	0.0000	-0.300	0.0000	7.1190
0.2860	1.0000	0.0000	0.0000	0.0000	-0.619	1.0000	0.7800	0.0000	20535
0.4800	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.4000	0.0000	54.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.9524	0.0000	0.0000	1.0000	190.48

Our results show that to best optimize our money and space we should have 54 three person houses and seven four person houses.

When running with different numbers for the zoning law equations, we often get just one type of house built and not that many of it. This is due to the large amount of space but a smaller budget. 20 million sounds like a lot of money, but I can only build around 63 three person houses. The data I used uses all the land and not just what is left to build on, because there is no data on that. Even so, there are still more than 189 homeless people in Albuquerque [14]. This is why one of the zoning law constraints is small. It hits the budget limit before even coming close to all the land being used. When running with a higher budget (where money is no longer an issue, so it hits the land constraints), it only wants to make three and four person houses, probably because they are the most space efficient.

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